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ABSTRACT

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A Continuation Ratio Model for Ordered Category Items

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Abstract

Continuation ratio logits are used to model the probabilities of obtaining ordered categories in a polytomously scored item. This model is an alternative to other models for ordered category items such as the graded response model and the generalized partial credit model. The discussion includes a theoretical development of the model, a description of special cases, maximum marginal likelihood estimation of the item parameters, and estimation of ability. Comparisons of the item response theory models for ordered category items are presented using empirical data.

Key words: item response theory, maximum marginal likelihood, multicategory logit model, polytomous response.

Introduction

When a free response item is scored in a dichotomous fashion, for example, either correct or incorrect, a single stage decision is performed in a sense that no further decisions will be made beyond the current decision to be taken. When a free response item is rated in a polytomous fashion, a multiple stage decision is required in which dependent decisions are made in tandem eventually. Note that the true multiple stage decision process arises only if a rater decides not to assign the extreme score or category to the free response in the first stage decision because the further decision must be made for such a situation.

Borrowing terms from the game theory (Luce & Raiffa, 1957), the particular alternative chosen by a rater at a given decision point is called the "choice," and the totality of choices available to a rater at the decision point constitutes the "move." A sequence of choices, one following another until the rating or scoring of an item is complete, can be called a "play" (see also Kim, 1992; Taha, 1987). The play or the rating process for a given item can be depicted with a connected graph consists of a collection of nodes and branches between pairs of nodes. Human information processing to reach a decision, or to assign a category to an item in the current situation, seems to require performing of multiple loops in a certain stage and even the repetition of the whole decision process for a given item. For the sake of brevity, a connected graph without any loop of branches or any reiterating move, called the decision tree, is used to depict the play. The single stage decision and the multiple stage decisions are presented in Figure 1 (i.e., Figures 1-a to 1-d). The decision trees reflect the sequential nature of scoring. Each decision point is denoted as a circle and the chance events with respective but dependent probabilities are denoted as squares in Figure 1. Although other representations of the decision process is possible [e.g., see Figure 2 (i.e., Figures 2-a and 2-b)], the representation in Figure 1 is assumed to facilitate the decision making process and to be modeled in this study.

Insert Figure 1 and Figure 2 about here

The decision trees for the multiple stage scoring depicted in Figure 1 involve in a set of dependent events. The model for the ordered choices ought to reflect the joint probabilities and must take into account the conditional probabilities that characterize the dependence. The model for ordered category items to be described is called a continuation ratio model.

The continuation ratio model was developed to handle polytomous response variables in logit models (Fienberg, 1977, 1980; Fienberg & Mason, 1978). Comparisons among the continuation ratio model and other models for ordered polytomous responses in logistic regression and loglinear modeling can be found in Agresti (1990, 1996), Clogg and Shihadeh (1994), and O'Connell (2000). In the item response theory field, Mellenbergh (1995) presented conceptual notes on models for discrete polytomous item responses and indicated that the continuation ratio model could be considered as a special case of the Bock's (1972) model. Tutz (1990, 1997) and Verhelst, Glas, and de Vries (1997) proposed special cases of the continuation ratio model called the sequential item response model and the steps model, respectively. Some theoretical considerations of the continuation ratio model and other models for polytomous items were presented in Hemker, van der Ark, and Sijtsma (2001).

The Model

Let the Y_{ij} is the random variable designates the rating or scored item response of individual i to item j. The model considered in this paper assumes that the manifestation of Y_{ij} or the probability of Y_{ij} to be a specific value depends on a person's latent ability θ_i and a vector-valued item characteristics ξ_j.

The probability that $y_{ij} = k$ given ability θ_i and item parameter ξ_j is

$$\text{Prob}\,(y_{ij} = k|\theta_i, \xi_j) = P_{jk}(\theta_i) = \frac{\exp\left[-a_{jk}(\theta_i - b_{jk})\right]}{\prod_{h=1}^{k}\left\{1 + \exp\left[-a_{jh}(\theta_i - b_{jh})\right]\right\}} \tag{1}$$

for $k = 1, \ldots, K_j - 1$ and

$$P_{jK_j}(\theta_i) = 1 - \sum_{k=1}^{K_j-1} P_{jk}(\theta_i) = \frac{1}{\prod_{h=1}^{K_j-1}\left\{1 + \exp\left[-a_{jh}(\theta_i - b_{jh})\right]\right\}} \tag{2}$$

for $k = K_j$.

Under the assumption of conditional independence, the probability of a response vector $y_i = (y_{i1}, \ldots, y_{iJ})$, is given as

$$\text{Prob}\,(y_i|\theta_i, \xi) = p(y_i|\theta_i, \xi_1, \ldots, \xi_J) = \prod_{j=1}^{J} P_{jk}(\theta_i), \tag{3}$$

and the joint probability of the response vectors of a sample of I subjects is given as

$$\text{Prob}\,(y|\theta,\xi) = p(y_1,\ldots,y_I|\theta_1,\ldots,\theta_I,\xi) = \prod_{i=1}^{I}\prod_{j=1}^{J} P_{jk}(\theta_i). \tag{4}$$

When the above joint probability is considered as a function of unknown parameters ξ and θ, we call it the likelihood L. Inference of the values of unknown parameters from observed data can be accomplished by maximizing the likelihood or its modifications with respect to the unknown parameters.

Estimation

There are several estimation procedures to obtain parameter estimates in the continuation ratio model. Among those, the marginal estimation based on Bock and Aitkin (1981) can be considered to be the de facto standard procedure for the estimation of item parameters. In order to relate the continuation ratio model under item response theory and under the generalized linear model and in order to link other estimation methods used in psychometrics that involve the ability parameters, three procedures are presented; the estimation with known ability, the joint estimation, and the marginal estimation. Also presented are, in conjuction with the marginal estimation, the EM algorithm (i.e., the detailed E-step and M-step), model fit statistics, characterizing ability, and the information function.

The Estimation with Known Ability

The estimation with know ability is also known as the bioassay solution (see Mislevy & Bock, 1984). In the estimation with known ability, the θ's are known quantities, determined by the experimenter. It is instructive to consider the estimation with known ability first so that we may note similarities and differences with the other estimation procedures. Because the abilities are known and grouping is employed, the subscript l is used.

Let, for $k = 1,\ldots,K_j - 1$,

$$P^*_{jk}(\theta_l) = \frac{1}{1+\exp[-a_{jk}(\theta_l - b_{jk})]}. \tag{5}$$

Then,

$$P_{jk}(\theta_l) = \begin{cases} 1 - P^*_{jk}(\theta_l) & \text{if } k = 1 \\ \prod_{h=1}^{k-1} P^*_{jh}(\theta_l)[1 - P^*_{jk}(\theta_l)] & \text{if } k = 2,\ldots,K_j - 1 \\ \prod_{h=1}^{K_j-1} P^*_{jh}(\theta_l) & \text{if } k = K_j. \end{cases} \tag{6}$$

If we let $\prod_{h=1}^{0} P_{jh}^*(\theta_l) = 1$ and $P_{jK_j}^*(\theta_l) = 0$, then, without loss of generality,

$$P_{jk}(\theta_l) = \prod_{h=1}^{k-1} P_{jh}^*(\theta_l)[1 - P_{jk}^*(\theta_l)]. \tag{7}$$

Also let N_{jl} is the number of subjects with ability θ_l to respond item j and r_{jkl} is the number of subjects with ability θ_l with response k to item j. Note that $N_{jl} = \sum_k r_{jkl}$ and $I = \sum_l N_{jl}$. Then,

$$L = \prod_{l=1}^{L}\prod_{j=1}^{J} \frac{N_{jl}!}{r_{j1l}! r_{j2l}! \cdots r_{jK_jl}!} P_{j1}(\theta_l)^{r_{j1l}} P_{j2}(\theta_l)^{r_{j2l}} \cdots P_{jK_j}(\theta_l)^{r_{jK_jl}} = \prod_{l=1}^{L}\prod_{j=1}^{J} \frac{N_{jl}}{\prod_{k=1}^{K_j} r_{jkl}} \prod_{k=1}^{K_j} P_{jk}(\theta_l)^{r_{jkl}}. \tag{8}$$

The right part of the likelihood can be seen as a multinomial distribution $M(n, \{r_k\}; \{p_k\})$ (Agresti, 1990, p. 38), characterized by the sample size n, observed counts $\{r_k\}$, and cell probabilities $\{p_k\}$, where $n = N_{jl}$, $r_k = r_{jkl}$, and $p_k = P_{jk}(\theta_l)$. The likelihood can be written as

$$L = \prod_{l=1}^{L}\prod_{j=1}^{J} M(N_{jl}, \{r_{jkl}\}; \{P_{jk}(\theta_l)\}). \tag{9}$$

The maximum likelihood solution for the item parameters yields the values that maximize the likelihood for given data.

It can be noted that the likelihood can be written as

$$\begin{aligned} L \;=\; & \prod_{l=1}^{L}\prod_{j=1}^{J} \binom{N_{jl}}{r_{j1l}} [1 - P_{j1}^*(\theta_l)]^{r_{j1l}} P_{j1}^*(\theta_l)^{N_{jl} - r_{j1l}} \times \\ & \binom{N_{jl} - r_{j1l}}{r_{j2l}} [1 - P_{j2}^*(\theta_l)]^{r_{j2l}} P_{j2}^*(\theta_l)^{N_{jl} - r_{j1l} - r_{j2l}} \times \\ & \vdots \\ & \binom{N_{jl} - r_{j1l} - \cdots - r_{j(K_j-2)l}}{r_{j(K_j-1)l}} [1 - P_{j(K_j-1)}^*(\theta_l)]^{r_{j(K_j-1)l}} P_{j(K_j-1)}^*(\theta_l)^{N_{jl} - r_{j1l} - \cdots - r_{j(K_j-1)l}}, \end{aligned} \tag{10}$$

where

$$1 - P_{jk}^*(\theta_l) = \frac{P_{jk}(\theta_l)}{1 - P_{j1}(\theta_l) - \cdots - P_{j(k-1)}(\theta_l)} = \frac{P_{jk}(\theta_l)}{1 - \sum_{h=1}^{k-1} P_{jh}(\theta_l)}, \tag{11}$$

that is,

$$\begin{aligned} 1 - P_{j1}^*(\theta_l) &= P_{j1}(\theta_l), \\ 1 - P_{j2}^*(\theta_l) &= \frac{P_{j2}(\theta_l)}{1 - P_{j1}(\theta_l)}, \end{aligned}$$

$$\vdots$$

$$1 - P^*_{j(K_j-1)}(\theta_l) = \frac{P_{j(K_j-1)}(\theta_l)}{1 - P_{j1}(\theta_l) - \cdots - P_{j(K_j-2)}(\theta_l)}.$$

If $B(n, r; p)$ denotes the binomial distribution with n trials, r success, and the success probability p, then the part of the likelihood consists of $B[N_{jl}, r_{j1l}; 1 - P^*_{j1}(\theta_l)]$, $B[N_{jl} - r_{j1l}, r_{j2l}; 1 - P^*_{j2}(\theta_l)]$, $\ldots$, $B[N_{jl} - r_{j1l} - \cdots - r_{j(K_j-2)l}, r_{j(K_j-1)l}; 1 - P^*_{j(K_j-1)}(\theta_l)]$, and, consequently,

$$L = \prod_{l=1}^{L} \prod_{j=1}^{J} \prod_{k=1}^{K_j-1} B[N_{jl} - \sum_{h=1}^{k-1} r_{jhl}, r_{jkl}; 1 - P^*_{jk}(\theta_l)]. \tag{12}$$

Equivalently, we may work with the log likelihood for the maximization,

$$\begin{aligned} \log L \;=\; & \sum_{l=1}^{L} \sum_{j=1}^{J} \text{constant} + r_{j1l} \log[1 - P^*_{j1}(\theta_l)] + (N_{jl} - r_{j1l}) \log P^*_{j1}(\theta_l) + \\ & \text{constant} + r_{j2l} \log[1 - P^*_{j2}(\theta_l)] + (N_{jl} - r_{j1l} - r_{j2l}) \log P^*_{j2}(\theta_l) + \\ & \vdots \\ & \text{constant} + r_{j(K_j-1)l} \log[1 - P^*_{j(K_j-1)}(\theta_l)] + (N_{jl} - r_{j1l} - \cdots - r_{j(K_j-1)l}) \log P^*_{j(K_j-1)}(\theta_l). \end{aligned} \tag{13}$$

That is,

$$\log L = \text{constant} + \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{k=1}^{K_j-1} \left[r_{jkl} \log[1 - P^*_{jk}(\theta_l)] + \left(N_{jl} - \sum_{h=1}^{k} r_{jhl} \right) \log P^*_{jk}(\theta_l) \right]. \tag{14}$$

This expression is maximized by differentiating with respect to the item parameters and then finding the values of item parameters that make the derivatives zero. These are likelihood equations. For example, the partial derivative with respect to a_{j1} is

$$\frac{\partial \log L}{\partial a_{j1}} = \sum_{l=1}^{L} \left[N_{jl} - r_{j1l} - N_{jl} P^*_{j1}(\theta_l) \right] (\theta_l - b_{j1}). \tag{15}$$

Note that for an item with K_j possible response or scoring categories, there exist $2(K_j - 1)$ item parameters. The partial derivatives with respect to a_{jk} and b_{jk} are:

$$\frac{\partial \log L}{\partial a_{jk}} = \sum_{l=1}^{L} \left[\left(N_{jl} - \sum_{h=1}^{k} r_{jhl} \right) - \left(N_{jl} - \sum_{h=1}^{k-1} r_{jhl} \right) P^*_{jk}(\theta_l) \right] (\theta_l - b_{jk}) \tag{16}$$

$$\frac{\partial \log L}{\partial b_{jk}} = - \sum_{l=1}^{L} \left[\left(N_{jl} - \sum_{h=1}^{k} r_{jhl} \right) - \left(N_{jl} - \sum_{h=1}^{k-1} r_{jhl} \right) P^*_{jk}(\theta_l) \right] a_{jk} \tag{17}$$

If a vector of item parameters that solves these equations is unique and if the matrix of second derivatives of the log likelihood with regard to the item parameters is positive definite, then the resulting values are the maximum likelihood estimates of item parameters. The second derivatives are obtained as:

$$\frac{\partial^2 \log L}{\partial a_{jk}^2} = -\sum_{l=1}^{L} P_{jk}^*(\theta_l)[1 - P_{jk}^*(\theta_l)] \left(N_{jl} - \sum_{h=1}^{k-1} r_{jhl} \right) (\theta_l - b_{jk})^2 \tag{18}$$

$$\frac{\partial^2 \log L}{\partial a_{jk} b_{jk}} = \sum_{l=1}^{L} P_{jk}^*(\theta_l)[1 - P_{jk}^*(\theta_l)] \left(N_{jl} - \sum_{h=1}^{k-1} r_{jhl} \right) (\theta_l - b_{jk}) a_{jk} \tag{19}$$

$$\frac{\partial^2 \log L}{\partial b_{jk}^2} = -\sum_{l=1}^{L} P_{jk}^*(\theta_l)[1 - P_{jk}^*(\theta_l)] \left(N_{jl} - \sum_{h=1}^{k-1} r_{jhl} \right) a_{jk}^2 \tag{20}$$

Let

$$r'_{jkl} = N_{jl} - \sum_{h=1}^{k} r_{jhl} \tag{21}$$

and

$$N'_{jkl} = N_{jl} - \sum_{h=1}^{k-1} r_{jhl}. \tag{22}$$

Then, the partial derivatives with respect to a_{jk} and b_{jk} can be written as:

$$\frac{\partial \log L}{\partial a_{jk}} = \sum_{l=1}^{L} \left[r'_{jkl} - N'_{jkl} P_{jk}^*(\theta_l) \right] (\theta_l - b_{jk}) \tag{23}$$

$$\frac{\partial \log L}{\partial b_{jk}} = -\sum_{l=1}^{L} \left[r'_{jkl} - N'_{jkl} P_{jk}^*(\theta_l) \right] a_{jk} \tag{24}$$

The second derivatives can also be written as:

$$\frac{\partial^2 \log L}{\partial a_{jk}^2} = -\sum_{l=1}^{L} P_{jk}^*(\theta_l)[1 - P_{jk}^*(\theta_l)] N'_{jkl} (\theta_l - b_{jk})^2 \tag{25}$$

$$\frac{\partial^2 \log L}{\partial a_{jk} b_{jk}} = \sum_{l=1}^{L} P_{jk}^*(\theta_l)[1 - P_{jk}^*(\theta_l)] N'_{jkl} (\theta_l - b_{jk}) a_{jk} \tag{26}$$

$$\frac{\partial^2 \log L}{\partial b_{jk}^2} = -\sum_{l=1}^{L} P_{jk}^*(\theta_l)[1 - P_{jk}^*(\theta_l)] N'_{jkl} a_{jk}^2 \tag{27}$$

All second derivatives for parameters from different items are zero, so the solutions are independent from one item to another. The solutions can be found by Newton-Raphson method, carried out item by item, using

$$\hat{\xi}_j^{(t+1)} = \hat{\xi}_j^{(t)} - H_j^{-1} f_j, \tag{28}$$

where t designate iterations,

$$H_j = \left. \frac{\partial^2 \log L}{\partial \xi_j \partial \xi_j'} \right|_{\hat{\xi}_j^{(t)}}, \tag{29}$$

and

$$f_j = \left.\frac{\partial \log L}{\partial \xi_j}\right|_{\hat{\xi}_j^{(t)}}. \tag{30}$$

For large samples of examinees, the maximum likelihood estimates follow a multivariate normal distribution with means equal to the true values and dispersion matrix given by the block diagonal matrix

$$\Sigma = -\text{diag}\left(H_1^{-1}, H_2^{-1}, \ldots, H_J^{-1}\right). \tag{31}$$

The Joint Estimation

In the psychometric setting, the values of θ's are not known, and so the previous estimation with known ability cannot be applied in general. A variation called the joint estimation (or joint maximum likelihood estimation) has been implemented in a number of computer programs [e.g., LOGIST (Wingersky, Patrick, & Lord, 1999)]. In the joint estimation, the log likelihood is maximized, namely,

$$\log L = \sum_{i=1}^{I}\sum_{j=1}^{J} \log P_{jk}(\theta_i) = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K_j} y_{ijk} \log P_{jk}(\theta_i), \tag{32}$$

where

$$y_{ijk} = \begin{cases} 1 & \text{if } y_{ij} = k \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

and

$$P_{jk}(\theta_i) = \prod_{k=1}^{K_j} P_{jk}(\theta_i)^{y_{ijk}}. \tag{34}$$

Maximization is carried out with respect to the item parameters as well as θ_i parameters. The partial derivatives with respect to item parameters are obtained in a similar manner as in the previous estimation with known ability and not presented in detail. For example,

$$\frac{\partial \log L}{\partial a_{jk}} = \sum_{i=1}^{I}\sum_{k=1}^{K_j} y_{ijk} \frac{1}{P_{jk}(\theta_i)} \frac{\partial P_{jk}(\theta_i)}{\partial a_{jk}}. \tag{35}$$

There are I additional likelihood equations for the ability parameters,

$$\frac{\partial \log L}{\partial \theta_i} = \sum_{j=1}^{J}\sum_{k=1}^{K_j} y_{ijk} \frac{1}{P_{jk}(\theta_i)} \frac{\partial P_{jk}(\theta_i)}{\partial \theta_i} = \sum_{j=1}^{J}\sum_{k=1}^{K_j} y_{ijk} \frac{\partial \log P_{jk}(\theta_i)}{\partial \theta_i}. \tag{36}$$

The last term of the equation may not be easy to solve. We can use the relationship, however,

$$\log P_{jk}(\theta_i) = \sum_{h=1}^{k-1} \log P_{jh}^{*}(\theta_i) + \log[1 - P_{jk}^{*}(\theta_i)]. \tag{37}$$

For example, if we have an item with $K_j = 4$, then

$$\log P_{jk}(\theta_i) = \begin{cases} \log[1 - P^*_{j1}(\theta_i)] & \text{if } k = 1 \\ \log P^*_{j1}(\theta_i)] + \log[1 - P^*_{j2}(\theta_i)] & \text{if } k = 2 \\ \log P^*_{j1}(\theta_i) + \log P^*_{j2}(\theta_i) + \log[1 - P^*_{j3}(\theta_i)] & \text{if } k = 3 \\ \log P^*_{j1}(\theta_i) + \log P^*_{j2}(\theta_i) + \log P^*_{j3}(\theta_i) & \text{if } k = 4. \end{cases} \tag{38}$$

Hence,

$$\frac{\partial \log P_{jk}(\theta_i)}{\partial \theta_i} = \begin{cases} a_{j1} P^*_{j1}(\theta_i) & \text{if } k = 1 \\ -a_{j1}[1 - P^*_{j1}(\theta_i)] + a_{j2} P^*_{j2}(\theta_i) & \text{if } k = 2 \\ -a_{j1}[1 - P^*_{j1}(\theta_i)] - a_{j2}[1 - P^*_{j2}(\theta_i)] + a_{j3} P^*_{j3}(\theta_i) & \text{if } k = 3 \\ -a_{j1}[1 - P^*_{j1}(\theta_i)] - a_{j2}[1 - P^*_{j2}(\theta_i)] - a_{j3}[1 - P^*_{j3}(\theta_i)] & \text{if } k = 4 \end{cases} \tag{39}$$

and the second derivatives are

$$\frac{\partial^2 \log P_{jk}(\theta_i)}{\partial \theta_i^2} =$$

$$\begin{cases} -a^2_{j1} P^*_{j1}(\theta_i)[1 - P^*_{j1}(\theta_i)] & \text{if } k = 1 \\ -a^2_{j1} P^*_{j1}(\theta_i)[1 - P^*_{j1}(\theta_i)] - a^2_{j2} P^*_{j2}(\theta_i)[1 - P^*_{j2}(\theta_i)] & \text{if } k = 2 \\ -a^2_{j1} P^*_{j1}(\theta_i)[1 - P^*_{j1}(\theta_i)] - a^2_{j2} P^*_{j2}(\theta_i)[1 - P^*_{j2}(\theta_i)] - a^2_{j3} P^*_{j3}(\theta_i)[1 - P^*_{j3}(\theta_i)] & \text{if } k = 3 \\ -a^2_{j1} P^*_{j1}(\theta_i)[1 - P^*_{j1}(\theta_i)] - a^2_{j2} P^*_{j2}(\theta_i)[1 - P^*_{j2}(\theta_i)] - a^2_{j3} P^*_{j3}(\theta_i)[1 - P^*_{j3}(\theta_i)] & \text{if } k = 4. \end{cases} \tag{40}$$

Rather than solving for all item and ability parameters simultaneously, it is customary to maximize the log likelihood in cycles (i.e., the Birnbaum paradigm). The maximum likelihood ability estimate based on the Newton-Raphson method is

$$\hat{\theta}_i^{(t+1)} = \hat{\theta}_i^{(t)} - \left. \frac{\partial \log L / \partial \theta_i}{\partial^2 \log L / \partial \theta_i^2} \right|_{(t)} . \tag{41}$$

This joint estimation may have some problems (see Baker, 1992). The most serious one is that estimates of item parameters are inconsistent for tests of finite length (Andersen, 1972) even as the number of subjects increases without bound.

The Marginal Estimation

Suppose that it is reasonable to think of subjects as a random sample from a population in which ability is distributed in accordance with the density $g(\theta)$. Rather than obtaining a possibly unstable point estimate of θ_i given y_i, we could use the Bayes theorem to compute the entire posterior distribution of θ given y_i using

$$p(\theta | y_i) = \frac{p(y_i | \theta) g(\theta)}{\int_\Theta p(y_i | \theta) g(\theta) d\theta}. \tag{42}$$

Then, the discrete distribution to be used in the estimation of item parameters could be formed not by placing the data for an individual all at one point estimate but by distributing it across the ability scale in proportion to the posterior probability. The actual steps in the marginal estimation are similar to those in the joint estimation. The principal difference is in distributing each subject's data over the ability scale in proportion to the posterior probability rather than placing entire mass at one point. It is intuitively clear that this procedure is especially advantageous for short tests since the ability estimates will be most unstable, and be most wrong in assigning a single value for the unknown ability. Moreover, the posterior density exists for all response patterns, including those for which the maximum likelihood estimates are infinite. As test length increases, the joint estimation and the marginal estimation may become identical. The catch is that to compute $p(y_i|\theta)$, we have to know the item parameters. The marginal estimation, hence, is necessarily iterative like the joint estimation.

Under the usual assumption of conditional independence, the probability of response vector y_i from a subject with ability θ is given by

$$p(y_i|\theta, \xi) = \prod_{j=1}^{J} P_{jk}(\theta). \tag{43}$$

The probability of observing the response pattern from a subject selected at random from a population which θ is distributed in accordance with the density function $g(\theta)$ is given by the integral of the probability over the population

$$p(y_i|\xi) = \int p(y_i|\theta, \xi) g(\theta) d\theta. \tag{44}$$

This is the marginal probability of response pattern y_i with respect to the population density g, conditioned upon the item parameters. The marginal probability of a sample of response pattern is the product of the marginal probability over subjects and can be written as

$$L = \prod_{i=1}^{I} p(y_i|\xi) = \prod_{i=1}^{I} \int p(y_i|\theta, \xi) g(\theta) d\theta. \tag{45}$$

For given data, this is the likelihood function for the item parameters and the parameters of the population distribution. Assuming g to be known, the item parameters are estimated by maximizing the log of the marginal likelihood,

$$\log L = \sum_{i=1}^{I} \log p(y_i|\xi) = \sum_{i=1}^{I} \log \int p(y_i|\theta, \xi) g(\theta) d\theta. \tag{46}$$

This is done by taking the first derivatives with respect to the item parameters and setting the results to zero to produce the likelihood equations. Whether a solution is a maximum can be determined by examining the matrix of second derivatives of values of the likelihood in the neighborhood of the solution. The value of -2 times the likelihood can be obtained at each iteration, and the values should generally be decreasing (see Mislevy & Bock, 1984 for discussion of advantages and disadvantages of the marginal estimation).

The maximum marginal likelihood equations are needed. For example, the first derivative of a_{jk} is

$$
\begin{aligned}
\frac{\partial \log L}{\partial a_{jk}} &= \frac{\partial}{\partial a_{jk}}\left[\sum_{i=1}^{I} \log p(y_i|\xi)\right] \\
&= \sum_{i=1}^{I} \frac{1}{p(y_i|\xi)} \frac{\partial}{\partial a_{jk}} [p(y_i|\xi)] \\
&= \sum_{i=1}^{I} \frac{1}{p(y_i|\xi)} \frac{\partial}{\partial a_{jk}} \int p(y_i|\theta,\xi) g(\theta) d\theta \\
&= \sum_{i=1}^{I} \frac{1}{p(y_i|\xi)} \int \frac{\partial}{\partial a_{jk}} [p(y_i|\theta,\xi)]\, g(\theta) d\theta \\
&= \sum_{i=1}^{I} \int \left\{ \frac{\partial}{\partial a_{jk}} [\log p(y_i|\theta,\xi)] \right\} \left[\frac{p(y_i|\theta,\xi) g(\theta)}{p(y_i|\xi)} \right] d\theta \qquad (47) \\
&= \sum_{i=1}^{I} \int \left\{ \frac{\partial}{\partial a_{jk}} [\log p(y_i|\theta,\xi)] \right\} p(\theta|y_i,\xi) d\theta. \qquad (48)
\end{aligned}
$$

We see that the first derivative in the marginal estimation has the form of the first derivative in the estimation with known ability, integrated over the posterior density of θ given data.

Bock and Lieberman (1970) gave a numerical solution to the likelihood equations under the two-parameter normal ogive model. Their solution was based on a straightforward application of the Newton-Raphson method. The first derivatives were similar to the formula above. The values of the integral which cannot generally be solved analytically were obtained by quadratures. Also computed was an approximation of the expected matrix of second derivatives. In order to obtain the partial derivative for $\log p(y_i|\theta, \xi)$, let us use an indicator variable,

$$
u_{ijk} = \begin{cases} 1 & \text{if move } k \text{ occurred for subject } i \text{ and item } j \\ 0 & \text{otherwise,} \end{cases} \qquad (49)
$$

where $k = 1, \ldots, (K_j - 1)$. If a subject i has been administered item j and assigned score k $(k \neq K_j)$, then all u_{ijh}, $h = 1, \ldots, k$, are unity. If assigned K_j, then all u_{ijh}, $h = 1, \ldots, K_j - 1$, are unity. Note that

$$\frac{\partial \log L}{\partial a_{jk}} = \sum_{i=1}^{I} \int u_{ijk} \left[(1 - y_{ijk}) - P_{jk}^{*}(\theta) \right] (\theta - b_{jk}) p(\theta | y_i, \xi) d\theta \tag{50}$$

$$= \int \left[\sum_{i=1}^{I} u_{ijk}(1 - y_{ijk}) p(\theta | y_i, \xi) - P_{jk}^{*}(\theta) \sum_{i=1}^{I} u_{ijk} p(\theta | y_i, \xi) \right] (\theta - b_{jk}) d\theta. \tag{51}$$

In 1981, Bock and Aitkin reformulated the likelihood equations to produce a solution that avoids the computational problems of the Bock and Lieberman solution. The procedure is equivalent to solution by Dempster, Laird, and Rubin's (1977) generalized EM algorithm. The approximation to integration was done by numerical quadratures. Integration by quadrature replaces the problem of finding the area under a curve by the simpler problem of finding sum of the areas of a finite number of rectangles which approximate the area under the curve. The quadrature approximation for $p(y_i|\xi)$ yields

$$p(y_i|\xi) = \sum_{q=1}^{Q} p(y_i | X_q, \xi) A(X_q), \tag{52}$$

where the points at which the function is evaluated, $X_1, \ldots, X_Q$, are referred to as quadrature points, and associated with them are the quadrature weights $A(X_q)$. The weights take into account the height of the density function g in the neighborhood of the X's and the width of the rectangles. We may use the standard normal density for g (Stroud & Secrest, 1966). Using the quadratures, the likelihood equation for a_{jk} is

$$\frac{\partial \log L}{\partial a_{jk}} = \sum_{q=1}^{Q} \left[\sum_{i=1}^{I} u_{ijk}(1 - y_{ijk}) p(X_q | y_i, \xi) - P_{jk}^{*}(X_k) \sum_{i=1}^{I} u_{ijk} p(X_q | y_i, \xi) \right] (X_q - b_{jk}) \tag{53}$$

$$= \sum_{q=1}^{Q} \left[\bar{r}_{jkq} - \bar{N}_{jkq} P_{jk}^{*}(X_q) \right] (X_q - b_{jk}), \tag{54}$$

where

$$\bar{r}_{jkq} = \sum_{i=1}^{I} u_{ijk}(1 - y_{ijk}) p(X_q | y_i, \xi) \tag{55}$$

$$= \sum_{i=1}^{I} u_{ijk}(1 - y_{ijk}) \left[p(y_i|\xi) \right]^{-1} p(y_i | X_q, \xi) A(X_q) \tag{56}$$

$$= \sum_{i=1}^{I} \left[\frac{u_{ijk}(1 - y_{ijk}) p(y_i | X_q, \xi) A(X_q)}{\sum_{q=1}^{Q} p(y_i | X_q, \xi) A(X_q)} \right] \tag{57}$$

and

$$\bar{N}_{jkq} = \sum_{i=1}^{I} u_{ijk} p(X_q | y_i, \xi) \tag{58}$$

$$= \sum_{i=1}^{I} u_{ijk} \left[p(y_i|\xi)\right]^{-1} p(y_i|X_q,\xi)A(X_q) \tag{59}$$

$$= \sum_{i=1}^{I} \left[\frac{u_{ijk} p(y_i|X_q,\xi)A(X_q)}{\sum_{q=1}^{Q} p(y_i|X_q,\xi)A(X_q)} \right]. \tag{60}$$

These are the expected number of subjects and subjects assigned category higher than k to item j at quadrature point q, given the population density g and the item parameters. Note that $\bar{r}_{jkq}$ and $\bar{N}_{jkq}$ depend on the $p(\theta|y_i,\xi)$ terms, which in tern depend on the unknown item parameters. This means the likelihood equations are implicit, and must be solved iteratively. In analogy to the EM algorithm, each iteration can be broken into two steps. The E-step evaluates $\bar{r}_{jkq}$ and $\bar{N}_{jkq}$ for all j, k, and q for provisional values of the item parameters. The M-steps solves the likelihood equations of the item parameters with the $\bar{r}_{jkq}$'s and $\bar{N}_{jkq}$'s treated as known values.

For a given item, with $\bar{r}_{jkq}$ and $\bar{N}_{jkq}$ taken as known the likelihood equations do not depend on the item parameters of any other item. The M-step consists of independent solutions to each of the J items in and of itself. Computation is very reduced from the Bock and Lieberman solution since each item solution can be accomplished by the Newton-Raphson method to find the item parameters alone, just as in the estimation with known ability. The Hessian produced in each item's M-step can be used to produce approximate standard errors for the item parameters. These M-step standard errors tend to be too small, however, because they do not take into account the dependence among items in the full likelihood solution nor the fact that $\bar{r}_{jkq}$'s and $\bar{N}_{jkq}$'s are estimated rather than known quantities. The full information matrix may be empoyed in the Fisher-scoring solution to obtain more precise standard errors (see Mislevy & Bock, 1990).

Note that the density function g may be approximated together with item parameters (see Mislevy, 1984; Mislevy & Bock, 1984)

The E-Step

The Bock and Aitkin algorithm is based on a discrete representation of $g(\theta)$ and the integrand of $p(y_i|\xi)$, both continuous densities, over Q quadrature points, X_q, with $q = 1, \ldots, Q$. Such a discrete representation of the continuous densities may be made arbitrarily close to continuous reality by choosing Q large, just as numerical integration may be made arbitrarily accurate by using sufficient quadrature points. However, large values of Q slow the computations (e.g., we may use $Q = 10$).

Under the assumption that the population is composed of individuals who are members of Q discrete classes with values $X_1, \ldots, X_Q$ on the latent variable, complete data sufficient statistics for the estimation of item parameters ξ_j for item j would consist of a table of counts $\{r_{jkq}\}$, in which each element r_{jkq} is the number of individuals in class X_q selecting or assigned response greater than k on item j. So the E-step of the Bock and Aitkin algorithm consists of the expected values of the r_{jkq}, conditioned on the data and the current provisional estimates of the item parameters, as well as the expected value of the N_{jkq}, that is, the number of individuals in class X_q.

Using the provisional estimates of the item parameters for each item, we can compute the expected values of the r_{jkq} as

$$E(r_{jkq}|y, \hat{\xi}) = \bar{r}_{jkq} = \sum_{i=1}^{I} u_{ijk}(1 - y_{ijk}) p(X_q|y_i, \hat{\xi}) \tag{61}$$

$$= \sum_{i=1}^{I} u_{ijk}(1 - y_{ijk}) \left[p(y_i|\hat{\xi})\right]^{-1} p(y_i|X_q, \hat{\xi}) A(X_q) \tag{62}$$

$$= \sum_{i=1}^{I} \left[\frac{u_{ijk}(1 - y_{ijk}) p(y_i|X_q, \hat{\xi}) A(X_q)}{\sum_{q=1}^{Q} p(y_i|X_q, \hat{\xi}) A(X_q)}\right] \tag{63}$$

and

$$E(N_{jkq}|y, \hat{\xi}) = \bar{N}_{jkq} = \sum_{i=1}^{I} u_{ijk} p(X_q|y_i, \hat{\xi}) \tag{64}$$

$$= \sum_{i=1}^{I} u_{ijk} \left[p(y_i|\hat{\xi})\right]^{-1} p(y_i|X_q, \hat{\xi}) A(X_q) \tag{65}$$

$$= \sum_{i=1}^{I} \left[\frac{u_{ijk} p(y_i|X_q, \hat{\xi}) A(X_q)}{\sum_{q=1}^{Q} p(y_i|X_q, \hat{\xi}) A(X_q)}\right], \tag{66}$$

where

$$p(y_i|\hat{\xi}) = \sum_{q=1}^{Q} p(y_i|X_q, \hat{\xi}) A(X_q).$$

Note that, while $\bar{r}_{jkq}$ and $\bar{N}_{jkq}$ are computed in a loop over the observed response patterns, the values $P_{jk}(\theta)$ for each item are required only for a fixed set of Q values of X_q. If those values are placed in a table before the E-step is begun, the computations involved in $\bar{r}_{jkq}$ and $\bar{N}_{jkq}$ are limited to table look-up, multiplication, and addition. The E-step yields a set of J $(K_j - 1) \times Q$ tables of non-integral artificial counts which are used as data in the M-step.

The M-Step

The M-step consists of maximum likelihood estimation of the parameters ξ_j for all items $j = 1, \ldots, J$, using the tables of expected values $\bar{r}_{jkq}$ and $\bar{N}_{jkq}$ as data. It is simply nonlinear regression.

In terms of the expected data, the loglikelihood is

$$\log L \propto \sum_{j=1}^{J} \sum_{k=1}^{K_j-1} \sum_{q=1}^{Q} \left[(\bar{N}_{jkq} - \bar{r}_{jkq}) \log[1 - P_{jk}^*(X_q)] + \bar{r}_{jkq} \log P_{jk}^*(X_q)] \right]. \quad (67)$$

Standard gradient methods may be used to maximize $\log L$ over the parameter space. The maximum likelihood estimates of the item parameter ξ_j are obtained where

$$\frac{\partial \log L}{\partial \xi_{jk}} = 0 \quad (68)$$

for all parameters in ξ_j.

Because

$$\frac{\partial \log L}{\partial \xi_{jk}} = \sum_{q=1}^{Q} \frac{\bar{r}_{jkq} - \bar{N}_{jkq} P_{jk}^*(X_q)}{P_{jk}^*(X_q)[1 - P_{jk}^*(X_q)]} \left(\frac{\partial P_{jk}^*(X_q)}{\partial \xi_{jk}} \right), \quad (69)$$

we have

$$\frac{\partial P_{jk}^*(X_q)}{\partial a_{jk}} = P_{jk}^*(X_q)[1 - P_{jk}^*(X_q)](X_q - b_{jk}) \quad (70)$$

and

$$\frac{\partial P_{jk}^*(X_q)}{\partial b_{jk}} = -P_{jk}^*(X_q)[1 - P_{jk}^*(X_q)] a_{jk}. \quad (71)$$

The second derivatives can also be written as:

$$\frac{\partial^2 \log L}{\partial a_{jk}^2} = -\sum_{l=1}^{L} P_{jk}^*(X_q)[1 - P_{jk}^*(X_q)] \bar{N}_{jkq} (X_q - b_{jk})^2 \quad (72)$$

$$\frac{\partial^2 \log L}{\partial a_{jk} b_{jk}} = \sum_{l=1}^{L} P_{jk}^*(X_q)[1 - P_{jk}^*(X_q)] \bar{N}_{jkq} (X_q - b_{jk}) a_{jk} \quad (73)$$

$$\frac{\partial^2 \log L}{\partial b_{jk}^2} = -\sum_{l=1}^{L} P_{jk}^*(X_q)[1 - P_{jk}^*(X_q)] \bar{N}_{jkq} a_{jk}^2 \quad (74)$$

Using the updated item parameters from the M-step, the sequence of the E-step and the M-step is repeated either the parameters stabilize or a fixed number of cycles is reached.

Model Fit Statistics

The fit of the model and subject ability distribution can be compared against the general multinomial alternative if the following conditions are met: (1) maximum likelihood estimation is employed. (2) All subjects respond to all items. (3) The number of item is less than or equal to 10. Under these conditions, the following statistic has approximately a chi-square distribution in large samples of subjects:

$$Q^2 = -2\sum_{l=1}^{L} r(y_l)\log\left[\frac{Np(y_l|\xi)}{r(y_l)}\right], \tag{75}$$

where summation runs over all potential response patterns y_l, $r(y_l)$ is the number of occurrences of a given pattern, and $p(y_l|\xi)$ is the marginal probability. Terms for patterns that have not been observed are set to zero. Degrees of freedom are given by the number of potential response patterns minus the number of parameters estimated minus one. The approximation to the chi-square distribution is poor when expected counts are small.

When strict maximum likelihood estimation is employed, likelihood ratio tests of model fit may also be obtained. Two types of tests are available; a comparison of nested models and a comparison of the model against the general multinomial alternative. The value of -2 times the log likelihood can be obtained after estimation cycle. For large samples of subjects values between the augmented model A and the more restrictive compact model C has approximately a chi-square distribution, that is,

$$Q^2 = (-2\log L_C) - (-2\log L_A), \tag{76}$$

where the degrees of freedom equal to the number of additional parameters estimated under the augmented model. This test may be used to compare the fit of nested model when separate estimation procedures are performed to provide maximum likelihood estimates of item parameters under both models.

Characterizing θ

The parameter estimation described above usually has the goal of "calibrating" a set of test items, after which the item parameters are to be taken as known and used to characterize θ for examinees who produce a particular response pattern y_i. (Note that another way to estimate the θ is to use the method of maximum likelihood as we have seen in the joint estimation earlier.)

Given a set of item parameters, the posterior density for θ is

$$p(\theta|y_i) = \prod_{j=1}^{J} P_{jk}(\theta)g(\theta). \tag{77}$$

If the model is correct, the above equation describes the distribution of examinees who respond with pattern y_i. For more than a few items, the posterior density is roughly Gaussian in shape, and so it may also be described by estimates of its location and spread. The procedure, called the maximum a posteriori (MAP) method, is to use the mode as an estimate of the location of the posterior density, where

$$\frac{\partial \log p(\theta|y_i)}{\partial\theta} = \sum_{j=1}^{J} \frac{\partial \log P_{jk}(\theta)}{\partial\theta} + \frac{\partial \log g(\theta)}{\partial\theta} = 0 \tag{78}$$

with the posterior variance approximated by the negative inverse of

$$E\left[\frac{\partial^2 \log p(\theta|y_i)}{\partial\theta^2}\right]. \tag{79}$$

The modal estimate is practical to compute as long as $g(\theta)$ is a reasonable function, and easy if $g(\theta)$ is normal. Because there is no guarantee that the posterior density is unimodal, potential multimodality may present problems for mechanical use of modal estimates.

It is fairly straight forward, however, to numerically integrate the posterior distribution to obtain its mean and variance. The mean has been called the expected a posterior (EAP) estimate of θ (see Bock & Mislevy, 1982). An advantage of the EAP procedure over modal estimation is that the derivative of $g(\theta)$ is not required. Therefore $g(\theta)$ may take any form describable as a histogram with finite variance.

The Information Function

The amount of information yielded by the item j at ability level is

$$I_j(\theta) = \sum_{k=1}^{K_j} I_{jk}(\theta)P_{jk}(\theta), \tag{80}$$

where the quantity $I_{jk}(\theta)P_{jk}(\theta)$ is the amount of information share of category k and $I_{jk}(\theta)$ is the information function of an item response category k defined as

$$I_{jk}(\theta) = -\frac{\partial^2 \log P_{jk}(\theta)}{\partial\theta^2} = -\frac{\partial}{\partial\theta}\left[\frac{P'_{jk}(\theta)}{P_{jk}(\theta)}\right] = -\frac{P''_{jk}(\theta)P_{jk}(\theta) - [P'_{jk}(\theta)]^2}{[P_{jk}(\theta)]^2}, \tag{81}$$

where $P'_{jk}(\theta) = \partial P_{jk}(\theta)/\partial\theta$ and $P''_{jk}(\theta) = \partial^2 P_{jk}(\theta)/\partial\theta^2$. Hence,

$$I_j(\theta) = \sum_{k=1}^{K_j} \left\{ \frac{[P'_{jk}(\theta)]^2}{P_{jk}(\theta)} - P''_{jk}(\theta) \right\}. \tag{82}$$

For the continuation ratio model,

$$I_j(\theta) = \sum_{k=1}^{K_j-1} \left\{ a_{jk}^2 P^*_{jk}(\theta)[1 - P^*_{jk}(\theta)] \prod_{h=1}^{k-1} P^*_{jh}(\theta) \right\}. \tag{83}$$

where $\prod_{h=1}^{0} P^*_{jh}(\theta) = 1$. The information function for a test with J item is

$$I(\theta) = \sum_{j=1}^{J} I_i(\theta). \tag{84}$$

An Example

Data

The 1998 Fall data of the Baseline version of the Georgia Kindergarten Assessment Program-Revised (GKAP-R) were analyzed. The Baseline version of the GKAP-R is a performance assessment rating instrument that consists of ten polytomously scored items with three ordered categories. For the purpose of the present example, four mathematics items were selected.

The full description of the GKAP-R can be obtained in the Georgia Department of Education web site:

`http://www.doe.k12.ga.us/curriculum/testing/gkap.asp`

A total of 105,731 students who did not have any omitted or unreached responses were used.

Parameter Estimation

The marginal estimation was carried out on four mathematics items form the test of the GKAP-R. Ten quadrature fractile points were used for ability integration during calculations.

After several cycles of the expected and maximization iterations, the item parameter estimates were stable to four significant figures. Goodness of fit for the model was assessed. The resulting chi-square value was 2067.44312. The chi-square has the degrees of freedom equal to the number of response patterns minus the number of parameters estimated minus one (i.e., 68=81-12-1).

Although the solution shows remarkably good fit, the chi-square is large mainly due to the large sample size. Taken into account with the quite large sample size used in the calibration, the agreement of the observed and estimated frequencies of the response patterns seems reasonably good.

Category Response Functions

Category response functions of the four items under the continuation ratio model are shown in Figure 3. Item parameter estimates from which the functions were constructed are presented in Table 1.

Insert Figure 3 and Table 1 about here

To make comparisons, category response functions of the items under the graded response model (Samejima, 1969, 1972, 1997), the generalized partial credit model (Muraki, 1992, 1997), and the partial credit model (Masters, 1982; Masters & Wright, 1997) are shown in Figures 4, 5, and 6, respectively. The respective sets of item parameter estimates are presented in Tables 2, 3, and 4. The computer program MULTILOG (Thissen, 1991) was used to obtain parameter estimates under the graded response model, the generalized partial credit model, and the partial credit model (see Childs & Chen, 1999, for the comparability of parameter estimates from polytomous models).

Insert Figures 4–7 and Tables 2–4 about here

For each of the items, the monotonic decreasing curve corresponds to the lowest category; the middle curve corresponds to the middle category; the monotonic increasing curve corresponds to the highest category. These indicate in each item that the examinees of indefinitely low ability will be assigned the lowest category and, conversely, that examinees of indefinitely high ability will be assigned the highest category.

Some extent, the form of the category response functions depicted in Figures 3 to 6, although obtained from the different values of item parameters based on the different models, are very similar except for the first and second categories of Item 4 (see Figure 7 for overall comparisons).

Information Functions

Item information shares of the three categories of Item 1, item information functions of the four items, and the test information function under the continuation ratio model are shown in Figure 8. The same sets of functions under the graded response model, the generalized partial credit model, and the partial credit model are shown in Figures 9, 10, and 11, respectively.

Insert Figures 8–12 about here

The overall information from the continuation ratio model can be compared with values obtained from the graded response model, the generalized partial credit model, and the partial credit model. The continuation ration model yielded a slightly higher information for the lower ability levels than other models (see Figure 12 for overall comparisons).

Ability Estimates

The expected a posteriori method was used to estimate the ability parameters. The ability estimates from the continuation ratio model are presented in Table 5. Table 5 also contains the corresponding standard deviations for the 81 response patterns. The ability estimates from the other models are presented in Tables 6 to 8. All ability estimates and the posterior standard deviations are very similar for all models. The differences in the estimates and the posterior standard deviations among the models occurred in mostly second and third decimal places.

Insert Tables 5–8 about here

Summary

An item response theory model which retains the sequential order of the item response categories is proposed for tests consisting of ordered category items. The model makes use of continuation ratio logits to describe the probability of assigning each category in terms of two item parameters and the ability parameter. Procedures based on the method of maximum likelihood are described for estimating item and ability parameters, and for testing goodness of fit of the model.

Three procedures of estimation item (and ability) parameters of the model are described. In the first estimation, the maximum likelihood estimates of item parameters are obtained assuming the ability parameters are known. In the joint estimation, item and ability estimates are obtained jointly by maximizing the likelihood function with respect to the item and ability parameters. In the marginal estimation, the likelihood function is integrated with respect to the ability distribution in order to obtain maximum marginal likelihood estimates of the item parameters. For the marginal estimation, the ability parameters are to be estimated subsequently assuming that the item parameter estimates are known values. The methods of maximum likelihood, the expected a posteriori, and the maximum a posteriori are available for the ability estimation.

An application of the maximum marginal likelihood method with the expected a posteriori estimation of ability is reported using data consisting of the responses of 105,731 examinees to a four item mathematics test of the Baseline version of the GKAP-R. Comparisons of the estimates of item and ability parameters from the continuation ratio model, the graded response model, and the generalized partial credit model as well as the partial credit model show the results of the models to be closely comparable. The results of the test of goodness of fit of the models show also similar comparability.

An information analysis is carried out to compare the precision of estimating ability under the models of polytomously scored items. The test information functions from the models show different patterns of precision along ability scale. For examinees below median ability, the continuation ratio model results in an increase in precision. For examinees above median ability, the continuation ratio model results in a slight decrease in precision.

The model considered in this paper can be applied to polytomous response items that have special characteristics. The polytomous responses are ordered instead of nominal. The categories or ordered levels of the response are assigned in a sequential manner. Not all polytomous ordered responses have such characteristics.

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Table 1

Continuation Ratio (CR) Model Item Parameter Estimates of the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

	CR Item Parameter Estimates			
Item	a_{j1}	b_{j1}	a_{j2}	b_{j2}
M1	2.37	−1.87	1.63	−1.77
M2	3.05	−0.87	2.04	−0.04
M3	2.17	−1.14	1.67	0.47
M4	2.03	−2.04	0.86	−1.12

Table 2

Graded Response (GR) Model Item Parameter Estimates of the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

Item	GR Item Parameter Estimates		
	a_j	b_{j1}	b_{j2}
M1	2.07	−1.97	−1.33
M2	2.62	−0.90	0.09
M3	1.98	−1.17	0.52
M4	1.23	−2.68	−0.71

Table 3

Generalized Partial Credit (GPC) Model Item Parameter Estimates of the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

	GPC Item Parameter Estimates					
Item	a_{j1}	a_{j2}	a_{j3}	c_{j1}	c_{j2}	c_{j3}
M1	−1.50	0.00	1.50	−2.29	−0.25	2.54
M2	−2.22	0.00	2.22	−1.21	0.58	0.64
M3	−1.74	0.00	1.74	−1.04	0.92	0.12
M4	−1.01	0.00	1.01	−1.88	0.44	1.45

Table 4

Partial Credit (PC) Model Item Parameter Estimates of the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

	PC Item Parameter Estimates					
Item	a_{j1}	a_{j2}	a_{j3}	c_{j1}	c_{j2}	c_{j3}
M1	−1.54	0.00	1.54	−2.33	−0.24	2.57
M2	−1.54	0.00	1.54	−0.89	0.35	0.54
M3	−1.54	0.00	1.54	−0.96	0.84	0.12
M4	−1.54	0.00	1.54	−2.41	0.65	1.76

Table 5

Expected A Posteriori (EAP) Ability Estimates and the Corresponding Posterior Standard Deviation (PSD) Using the Continuation Ratio (CR) Model for the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

Response	CR Model		
Pattern	EAP	PSD	Frequency
1111	−2.21	0.56	1971
1112	−1.71	0.47	1209
1113	−1.55	0.40	799
1121	−1.63	0.44	255
1122	−1.37	0.39	636
1123	−1.22	0.46	594
1131	−1.36	0.40	17
1132	−1.02	0.51	61
1133	−0.80	0.50	101
1211	−1.49	0.38	83
1212	−1.24	0.44	126
1213	−1.04	0.50	116
1221	−1.15	0.48	48
1222	−0.76	0.47	222
1223	−0.60	0.41	304
1231	−0.74	0.47	6
1232	−0.47	0.38	60
1233	−0.33	0.43	85
1311	−1.14	0.48	17
1312	−0.75	0.48	26
1313	−0.57	0.43	35
1321	−0.66	0.44	15
1322	−0.42	0.39	84
1323	−0.26	0.46	134
1331	−0.38	0.42	5
1332	−0.06	0.53	33
1333	0.19	0.56	101
2111	−1.73	0.47	697
2112	−1.47	0.33	935
2113	−1.36	0.37	736
2121	−1.42	0.35	268
2122	−1.16	0.47	741
2123	−0.95	0.51	723
2131	−1.14	0.48	15
2132	−0.75	0.48	77
2133	−0.57	0.43	120
2211	−1.32	0.39	147
2212	−0.98	0.50	291
2213	−0.77	0.48	278
2221	−0.87	0.50	181
2222	−0.57	0.38	1243
2223	−0.45	0.37	913
2231	−0.55	0.38	20
2232	−0.33	0.42	167
2233	−0.15	0.49	321
2311	−0.86	0.50	18
2312	−0.55	0.40	45
2313	−0.41	0.41	48
2321	−0.49	0.38	22
2322	−0.26	0.45	191
2323	−0.06	0.52	369
2331	−0.21	0.48	12
2332	0.18	0.55	86
2333	0.45	0.57	336
3111	−1.47	0.35	935
3112	−1.24	0.44	2258
3113	−1.04	0.50	2645
3121	−1.15	0.48	364
3122	−0.76	0.48	2423
3123	−0.58	0.44	3832
3131	−0.73	0.48	41
3132	−0.43	0.43	321
3133	−0.24	0.50	844
3211	−0.97	0.50	340
3212	−0.62	0.42	1393
3213	−0.48	0.40	2442
3221	−0.55	0.39	395
3222	−0.34	0.41	4886
3223	−0.16	0.49	10392
3231	−0.29	0.44	81
3232	0.06	0.54	1533
3233	0.32	0.56	5519
3311	−0.53	0.41	200
3312	−0.26	0.47	703
3313	−0.04	0.55	1372
3321	−0.22	0.47	331
3322	0.17	0.55	4714
3323	0.44	0.57	14402
3331	0.26	0.57	155
3332	0.72	0.61	3846
3333	1.07	0.68	23221

Table 6

Expected A Posteriori (EAP) Ability Estimates and Their Corresponding Posterior Standard Deviations (PSD) Using the Graded Response (GR) Model for the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

Response	GR Model		
Pattern	EAP	PSD	Frequency
1111	−2.18	0.60	1971
1112	−1.84	0.54	1209
1113	−1.61	0.50	799
1121	−1.59	0.49	255
1122	−1.40	0.46	636
1123	−1.14	0.53	594
1131	−1.38	0.56	17
1132	−1.14	0.56	61
1133	−0.80	0.56	101
1211	−1.45	0.47	83
1212	−1.27	0.49	126
1213	−0.99	0.53	116
1221	−0.97	0.52	48
1222	−0.79	0.48	222
1223	−0.58	0.40	304
1231	−0.66	0.49	6
1232	−0.53	0.44	60
1233	−0.30	0.48	85
1311	−1.20	0.58	17
1312	−0.96	0.57	26
1313	−0.62	0.55	35
1321	−0.62	0.53	15
1322	−0.47	0.49	84
1323	−0.17	0.54	134
1331	−0.13	0.60	5
1332	0.00	0.57	33
1333	0.37	0.55	101
2111	−1.70	0.46	697
2112	−1.53	0.38	935
2113	−1.38	0.42	736
2121	−1.37	0.43	268
2122	−1.22	0.48	741
2123	−0.94	0.52	723
2131	−1.12	0.53	15
2132	−0.92	0.53	77
2133	−0.63	0.50	120
2211	−1.25	0.47	147
2212	−1.08	0.51	291
2213	−0.80	0.48	278
2221	−0.79	0.47	181
2222	−0.65	0.40	1243
2223	−0.51	0.34	913
2231	−0.55	0.42	20
2232	−0.46	0.39	167
2233	−0.25	0.48	321
2311	−0.95	0.55	18
2312	−0.76	0.51	45
2313	−0.49	0.49	48
2321	−0.50	0.47	22
2322	−0.39	0.45	191
2323	−0.11	0.53	369
2331	−0.06	0.57	12
2332	0.06	0.55	86
2333	0.40	0.53	336
3111	−1.43	0.48	935
3112	−1.25	0.50	2258
3113	−0.96	0.54	2645
3121	−0.94	0.53	364
3122	−0.75	0.49	2423
3123	−0.52	0.45	3832
3131	−0.58	0.54	41
3132	−0.44	0.51	321
3133	−0.10	0.58	844
3211	−0.80	0.50	340
3212	−0.65	0.43	1393
3213	−0.47	0.40	2442
3221	−0.47	0.38	395
3222	−0.40	0.38	4886
3223	−0.18	0.49	10392
3231	−0.13	0.52	81
3232	−0.03	0.53	1533
3233	0.30	0.52	5519
3311	−0.42	0.54	200
3312	−0.28	0.53	703
3313	0.09	0.59	1372
3321	0.02	0.55	331
3322	0.13	0.54	4714
3323	0.46	0.52	14402
3331	0.58	0.57	155
3332	0.65	0.55	3846
3333	1.10	0.67	23221

Table 7

Expected A Posteriori (EAP) Ability Estimates and Their Corresponding Posterior Standard Deviations (PSD) Using the Generalized Partial Credit (GPC) Model for the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

Response	GPC Model		
Pattern	EAP	PSD	Frequency
1111	−2.15	0.59	1971
1112	−1.83	0.52	1209
1113	−1.60	0.44	799
1121	−1.66	0.46	255
1122	−1.47	0.42	636
1123	−1.27	0.47	594
1131	−1.33	0.45	17
1132	−1.09	0.51	61
1133	−0.83	0.49	101
1211	−1.56	0.43	83
1212	−1.38	0.44	126
1213	−1.16	0.50	116
1221	−1.23	0.49	48
1222	−0.97	0.52	222
1223	−0.72	0.46	304
1231	−0.78	0.48	6
1232	−0.59	0.40	60
1233	−0.43	0.40	85
1311	−1.11	0.51	17
1312	−0.84	0.50	26
1313	−0.63	0.42	35
1321	−0.68	0.44	15
1322	−0.51	0.39	84
1323	−0.35	0.43	134
1331	−0.40	0.41	5
1332	−0.19	0.50	33
1333	0.09	0.55	101
2111	−1.71	0.48	697
2112	−1.51	0.42	935
2113	−1.32	0.46	736
2121	−1.38	0.44	268
2122	−1.15	0.50	741
2123	−0.89	0.51	723
2131	−0.96	0.52	15
2132	−0.72	0.46	77
2133	−0.54	0.39	120
2211	−1.28	0.47	147
2212	−1.03	0.52	291
2213	−0.77	0.48	278
2221	−0.84	0.50	181
2222	−0.63	0.42	1243
2223	−0.47	0.39	913
2231	−0.51	0.39	20
2232	−0.35	0.43	167
2233	−0.12	0.52	321
2311	−0.73	0.46	18
2312	−0.55	0.39	45
2313	−0.39	0.41	48
2321	−0.44	0.40	22
2322	−0.25	0.48	191
2323	0.02	0.54	369
2331	−0.06	0.53	12
2332	0.24	0.55	86
2333	0.53	0.55	336
3111	−1.42	0.43	935
3112	−1.21	0.49	2258
3113	−0.95	0.52	2645
3121	−1.03	0.52	364
3122	−0.77	0.48	2423
3123	−0.58	0.40	3832
3131	−0.62	0.42	41
3132	−0.47	0.39	321
3133	−0.29	0.46	844
3211	−0.90	0.51	340
3212	−0.67	0.44	1393
3213	−0.50	0.39	2442
3221	−0.55	0.39	395
3222	−0.39	0.41	4886
3223	−0.18	0.50	10392
3231	−0.25	0.48	81
3232	0.02	0.54	1533
3233	0.32	0.54	5519
3311	−0.47	0.39	200
3312	−0.30	0.45	703
3313	−0.05	0.53	1372
3321	−0.13	0.52	331
3322	0.17	0.55	4714
3323	0.46	0.54	14402
3331	0.38	0.54	155
3332	0.69	0.57	3846
3333	1.08	0.67	23221

Table 8

Expected A Posteriori (EAP) Ability Estimates and Their Corresponding Posterior Standard Deviations (PSD) Using the Partial Credit (PC) Model for the Mathematics Baseline Section of the Georgia Kindergarten Assessment Program–Rivised

Response	PC Model		
Pattern	EAP	PSD	Frequency
1111	−2.19	0.58	1971
1112	−1.74	0.48	1209
1113	−1.46	0.41	799
1121	−1.74	0.48	255
1122	−1.46	0.41	636
1123	−1.15	0.50	594
1131	−1.46	0.41	17
1132	−1.15	0.50	61
1133	−0.74	0.49	101
1211	−1.74	0.48	83
1212	−1.46	0.41	126
1213	−1.15	0.50	116
1221	−1.46	0.41	48
1222	−1.15	0.50	222
1223	−0.74	0.49	304
1231	−1.15	0.50	6
1232	−0.74	0.49	60
1233	−0.43	0.44	85
1311	−1.46	0.41	17
1312	−1.15	0.50	26
1313	−0.74	0.49	35
1321	−1.15	0.50	15
1322	−0.74	0.49	84
1323	−0.43	0.44	134
1331	−0.74	0.49	5
1332	−0.43	0.44	33
1333	−0.05	0.55	101
2111	−1.74	0.48	697
2112	−1.46	0.41	935
2113	−1.15	0.50	736
2121	−1.46	0.41	268
2122	−1.15	0.50	741
2123	−0.74	0.49	723
2131	−1.15	0.50	15
2132	−0.74	0.49	77
2133	−0.43	0.44	120
2211	−1.46	0.41	147
2212	−1.15	0.50	291
2213	−0.74	0.49	278
2221	−1.15	0.50	181
2222	−0.74	0.49	1243
2223	−0.43	0.44	913
2231	−0.74	0.49	20
2232	−0.43	0.44	167
2233	−0.05	0.55	321
2311	−1.15	0.50	18
2312	−0.74	0.49	45
2313	−0.43	0.44	48
2321	−0.74	0.49	22
2322	−0.43	0.44	191
2323	−0.05	0.55	369
2331	−0.43	0.44	12
2332	−0.05	0.55	86
2333	0.45	0.59	336
3111	−1.46	0.41	935
3112	−1.15	0.50	2258
3113	−0.74	0.49	2645
3121	−1.15	0.50	364
3122	−0.74	0.49	2423
3123	−0.43	0.44	3832
3131	−0.74	0.49	41
3132	−0.43	0.44	321
3133	−0.05	0.55	844
3211	−1.15	0.50	340
3212	−0.74	0.49	1393
3213	−0.43	0.44	2442
3221	−0.74	0.49	395
3222	−0.43	0.44	4886
3223	−0.05	0.55	10392
3231	−0.43	0.44	81
3232	−0.05	0.55	1533
3233	0.45	0.59	5519
3311	−0.74	0.49	200
3312	−0.43	0.44	703
3313	−0.05	0.55	1372
3321	−0.43	0.44	331
3322	−0.05	0.55	4714
3323	0.45	0.59	14402
3331	−0.05	0.55	155
3332	0.45	0.59	3846
3333	1.06	0.69	23221

Figure Captions

Figure 1-a. A decision tree for the single stage decision.

Figure 1-b. A decision tree for the two stage sequential decision.

Figure 1-c. A decision tree for the three stage sequential decision.

Figure 1-d. A decision tree for the K stage sequential decision.

Figure 2-a. A decision tree for the three stage decision.

Figure 2-b. A decision tree for the three stage decision.

Figure 3. Category response functions of the four items under the continuation ratio model.

Figure 4. Category response functions of the four items under the graded response model.

Figure 5. Category response functions of the four items under the generalized partial credit model.

Figure 6. Category response functions of the four items under the partial credit model.

Figure 7. Category response functions of the four items under the continuation ratio model, the graded response model, the generalized partial credit model, and the partial credit model.

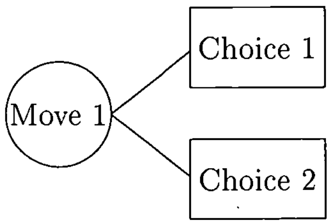
Figure 8. Item information shares of the three categories of Item 1, item infomation functions of the four items, and the test information function under the continuation ratio model.

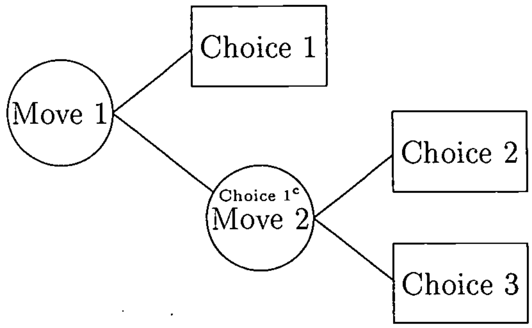
Figure 9. Item information shares of the three categories of Item 1, item infomation functions of the four items, and the test information function under the graded response model.

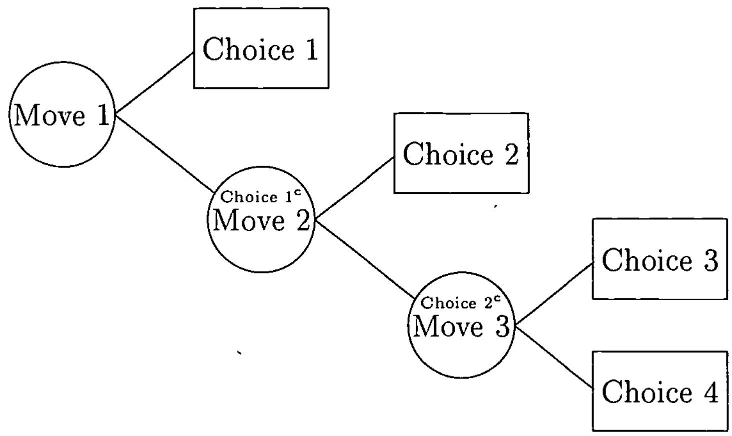
Figure 10. Item information shares of the three categories of Item 1, item infomation functions of the four items, and the test information function under the generalized partial credit model.

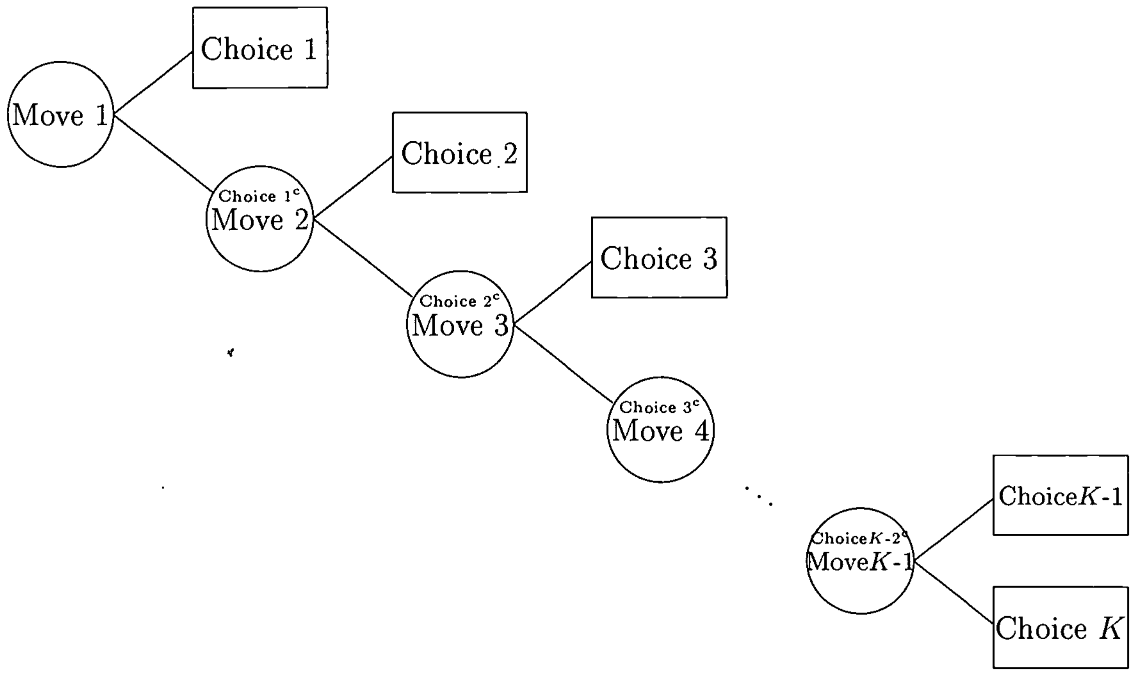
Figure 11. Item information shares of the three categories of Item 1, item infomation functions of the four items, and the test information function under the partial credit model.

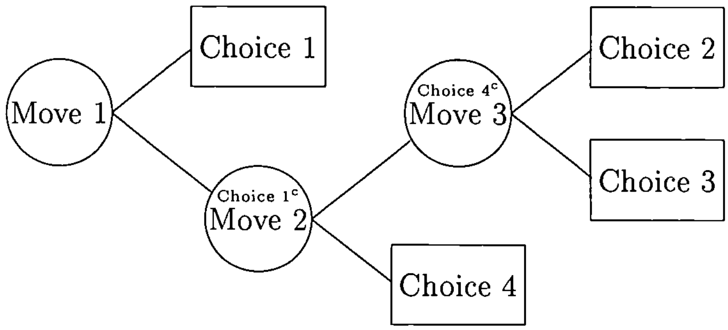
Figure 12. Item information shares of the three categories of Item 1, item infomation functions of the four items, and the test information function under the continuation ratio model, the graded response model, the generalized partial credit model, and the partical credit model.

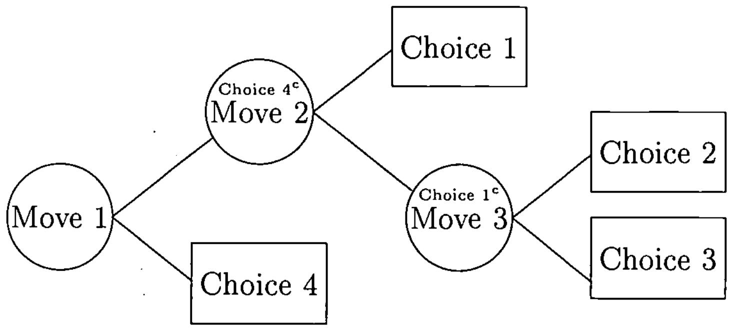

Move 1
Choice 1
Choice 2


Choice 1
Move 1
Choice 2
Choice 1c
Move 2
Choice 3

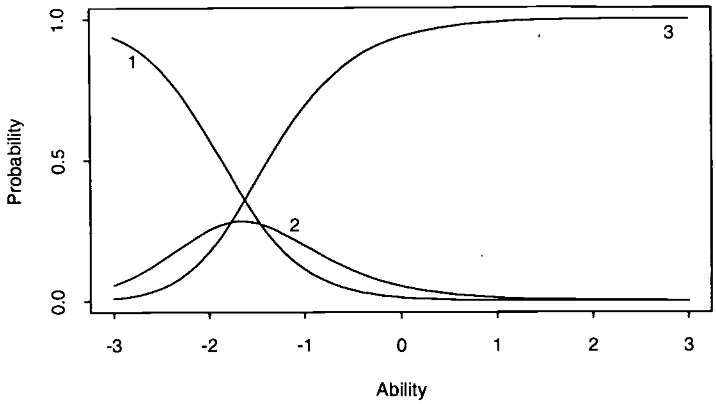

Move 1
Choice 1
Choice 1c
Move 2
Choice 2
Choice 2c
Move 3
Choice 3
Choice 4


Choice 1
Move 1
Choice 2
Choice 1^c
Move 2
Choice 3
Choice 2^c
Move 3
Choice 3^c
Move 4
Choice$K$-1
Choice$K$-2^c
Move$K$-1
Choice K

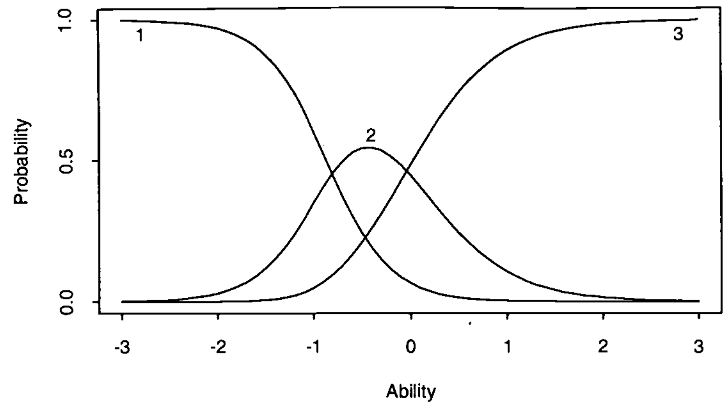

Choice 1
Move 1
Choice 1c
Move 2
Choice 4c
Move 3
Choice 2
Choice 3
Choice 4


Choice 1
Choice 4[c]
Move 2
Choice 2
Move 1
Choice 1[c]
Move 3
Choice 3
Choice 4

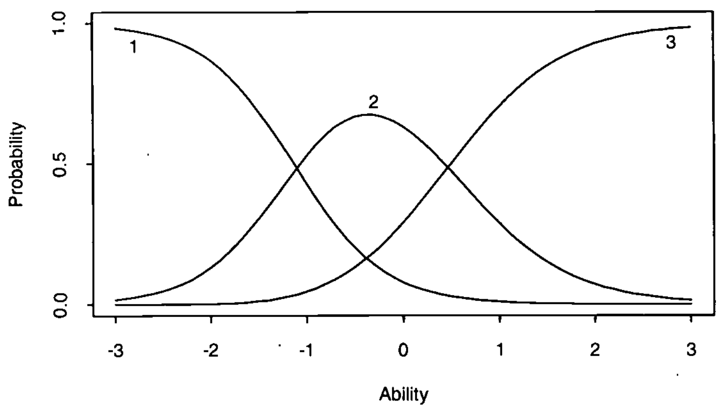
CR Category Response Functions of Item 1



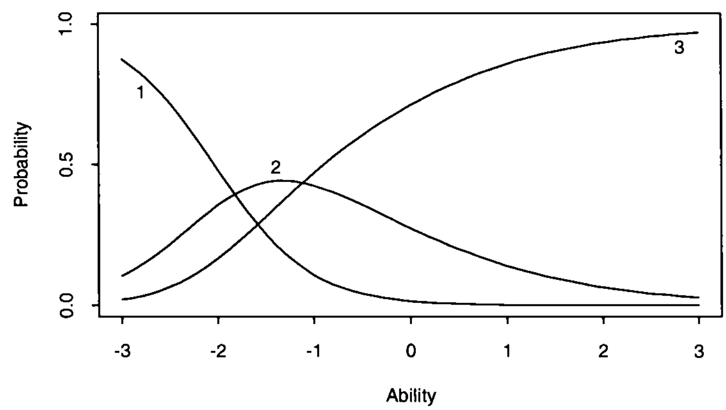
CR Category Response Functions of Item 2



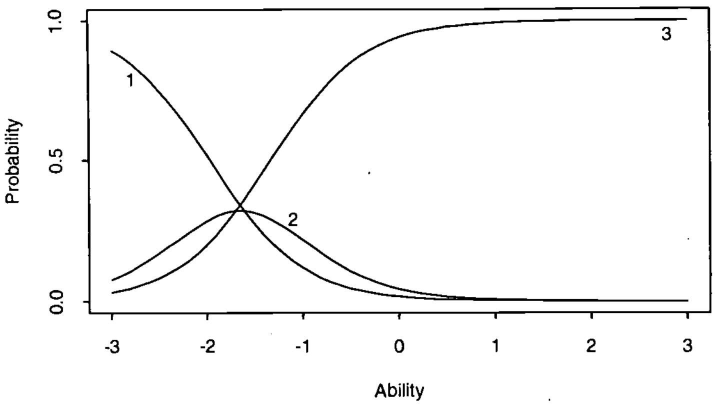
CR Category Response Functions of Item 3



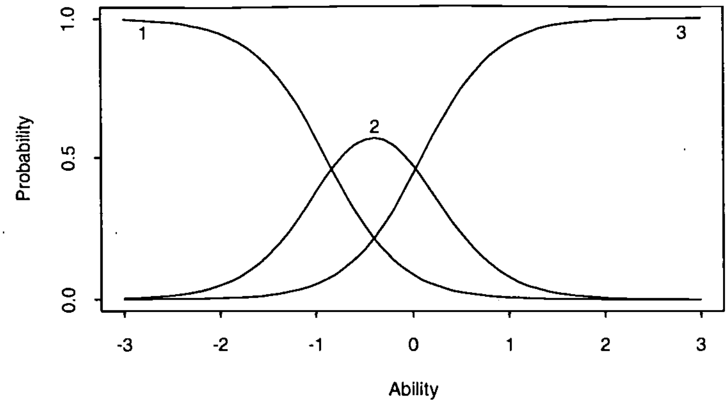
CR Category Response Functions of Item 4



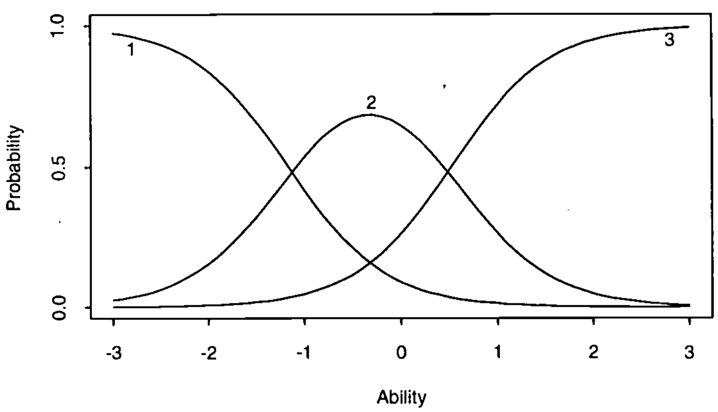
GR Category Response Functions of Item 1



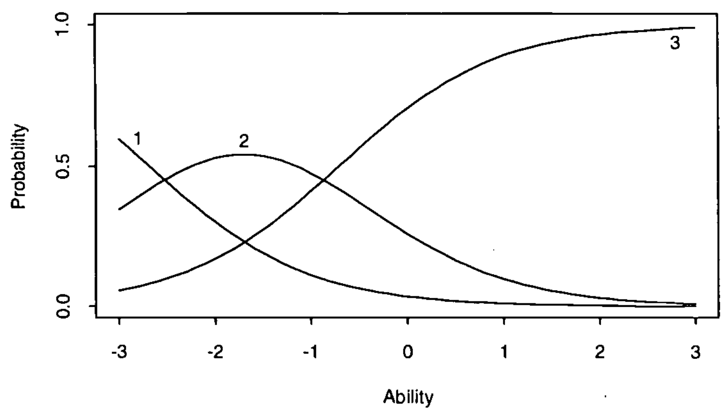
GR Category Response Functions of Item 2



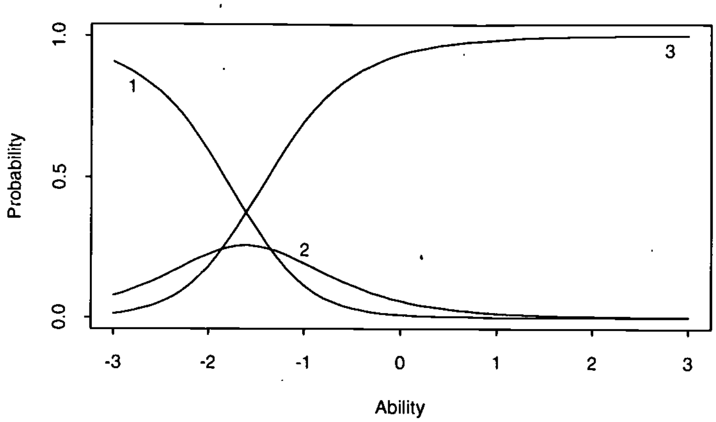
GR Category Response Functions of Item 3



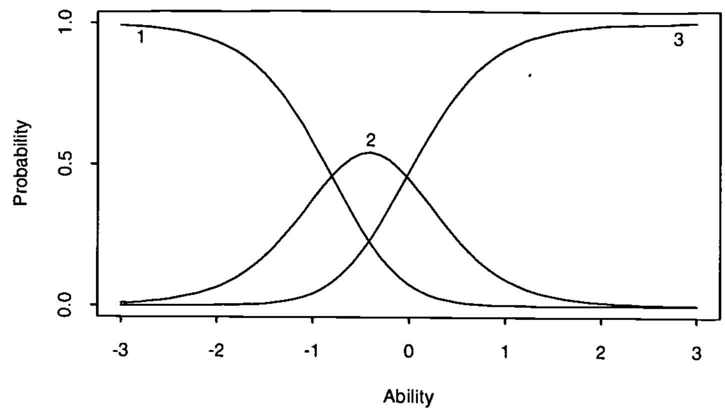
GR Category Response Functions of Item 4



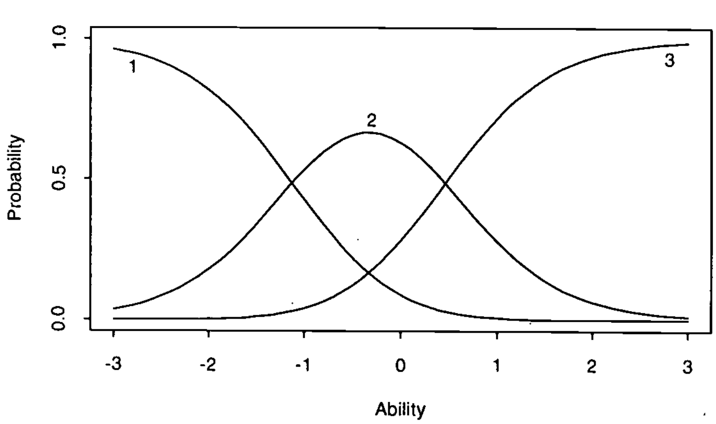
GPC Category Response Functions of Item 1



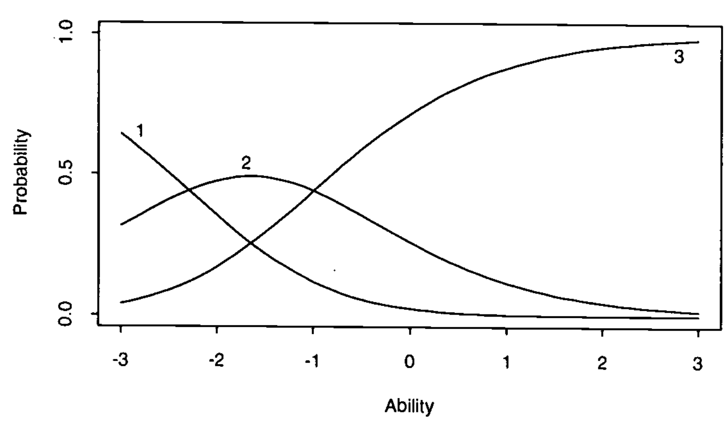
GPC Category Response Functions of Item 2



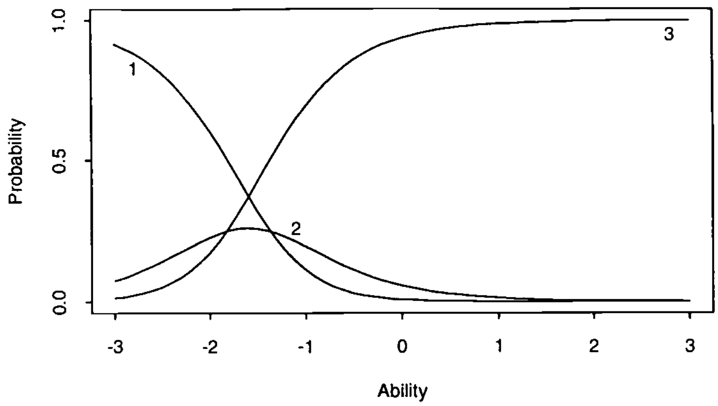
GPC Category Response Functions of Item 3



GPC Category Response Functions of Item 4



PC Category Response Functions of Item 1



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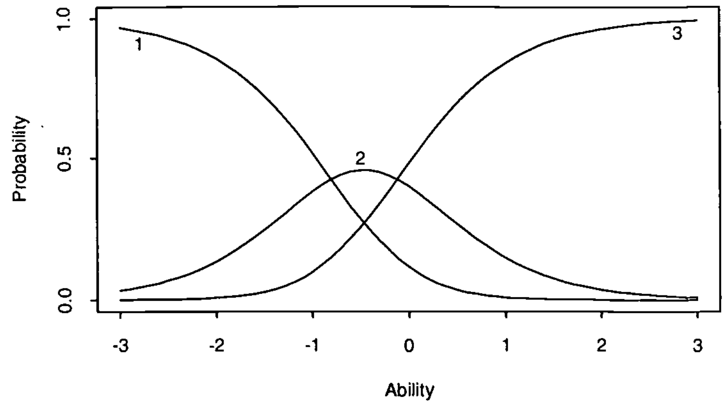

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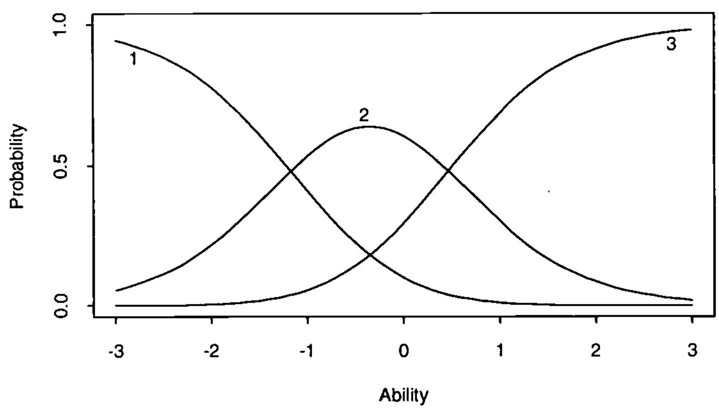

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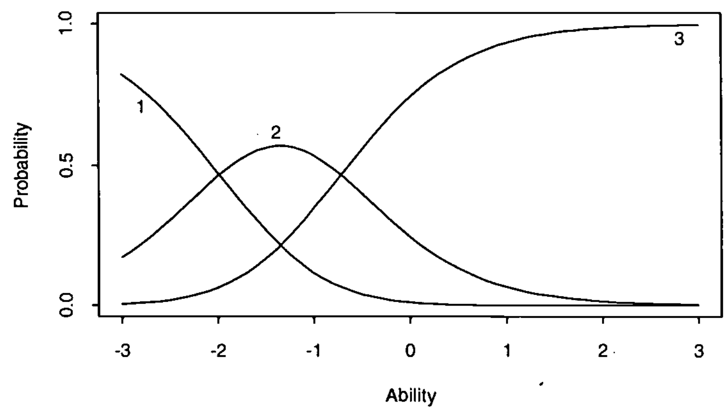

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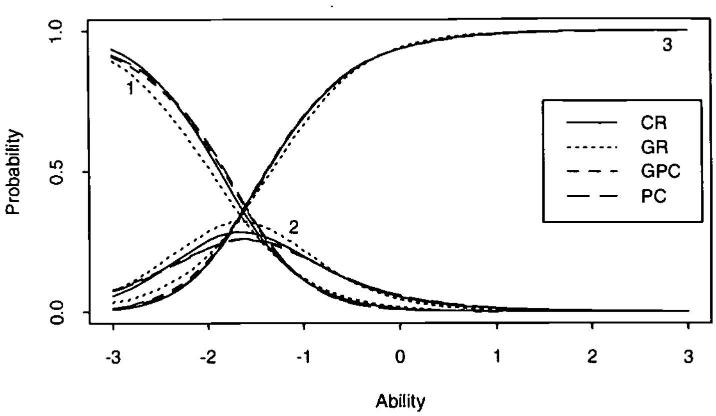

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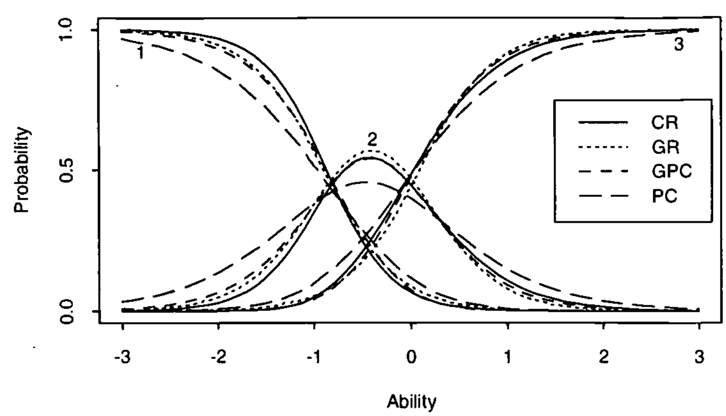

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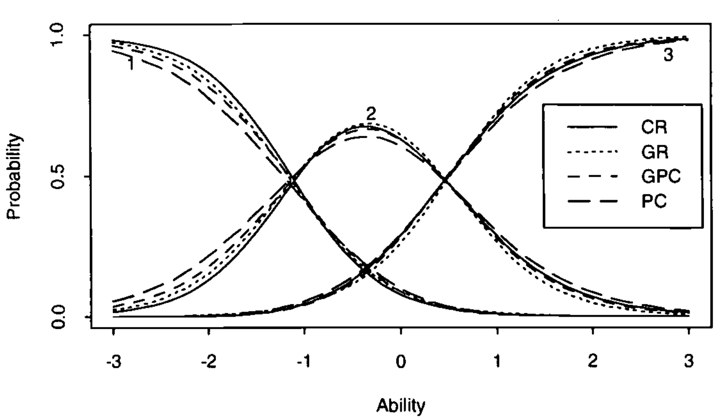

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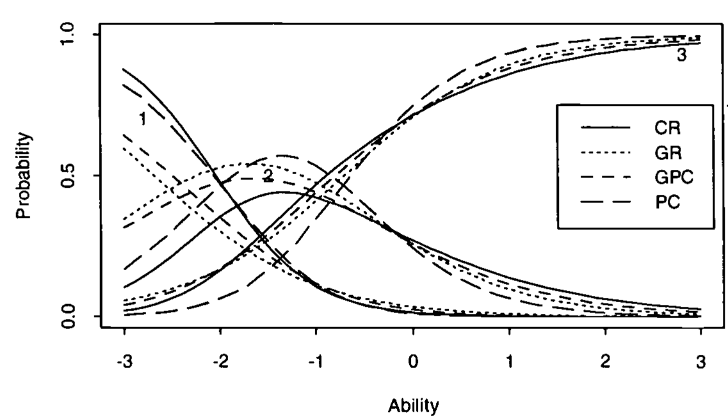

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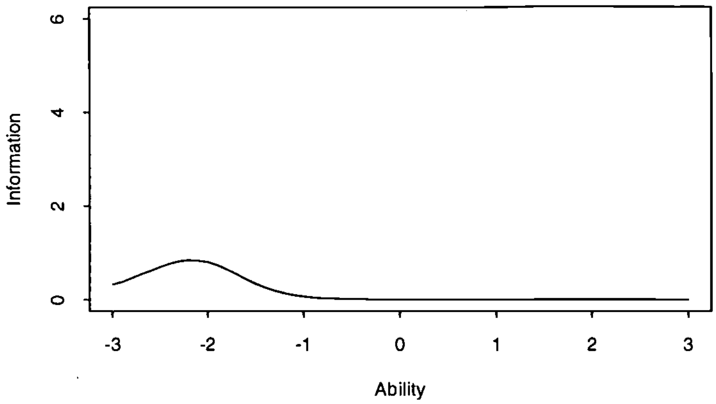

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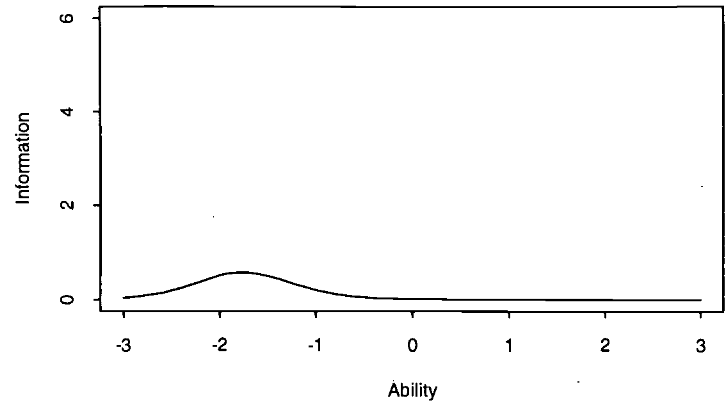

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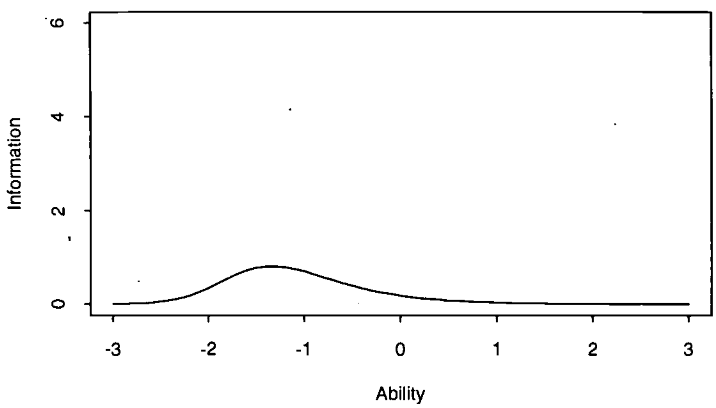

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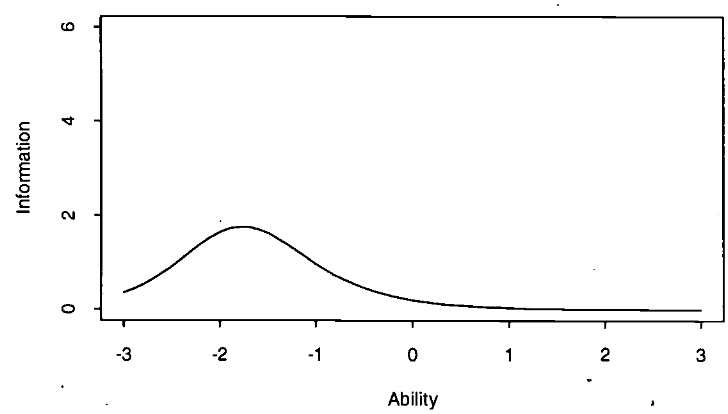

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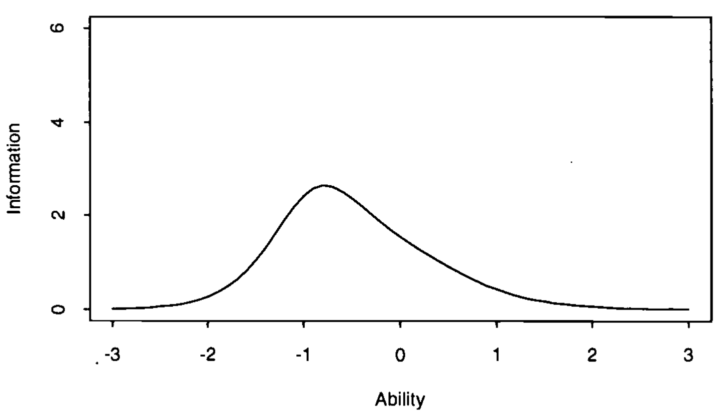

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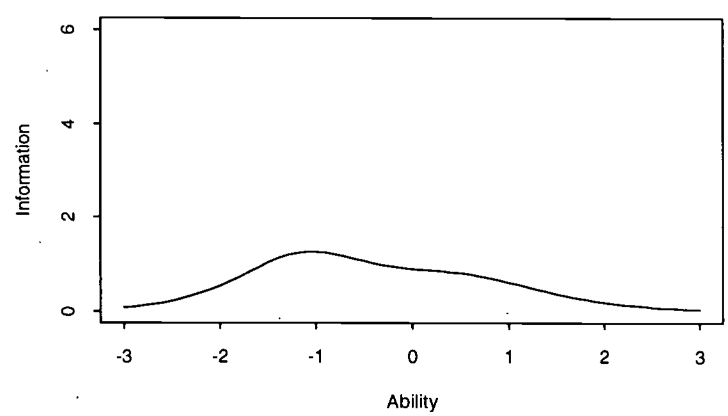

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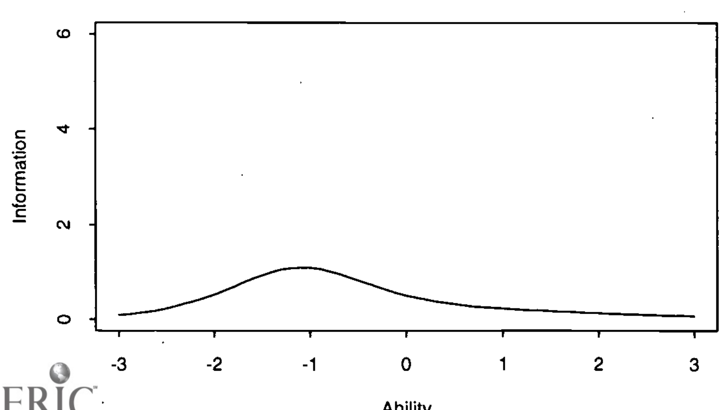

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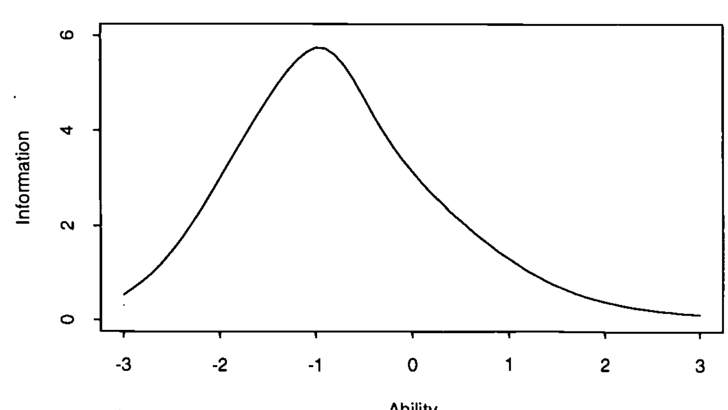

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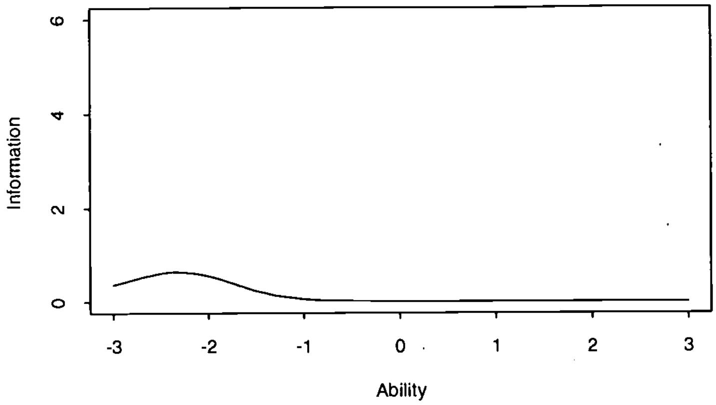

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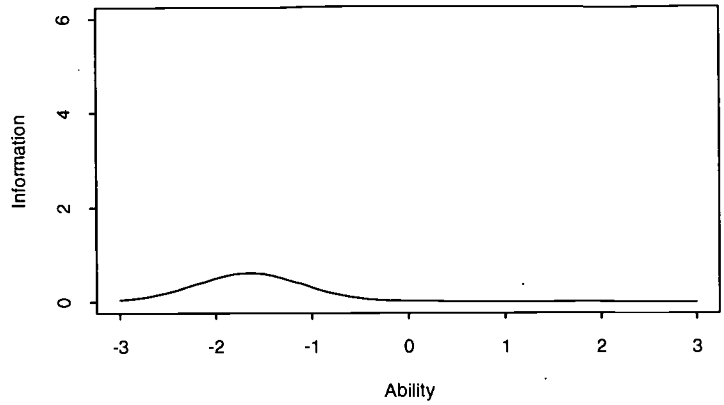

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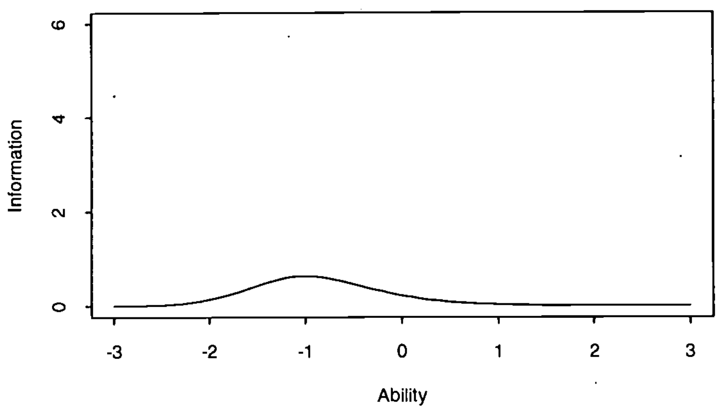

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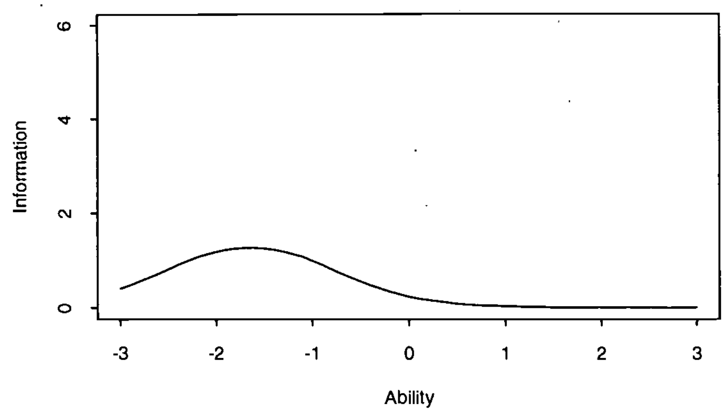

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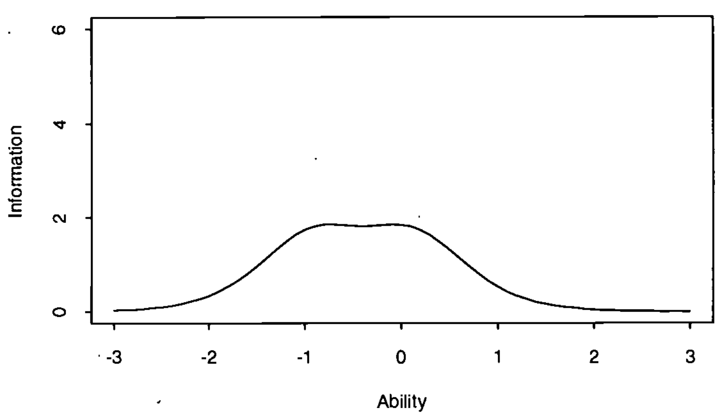

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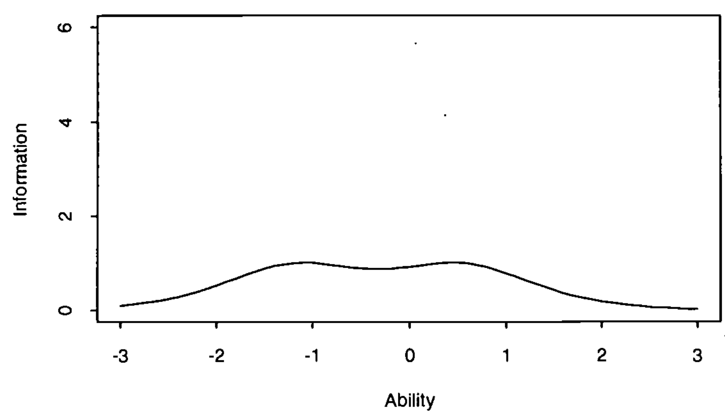

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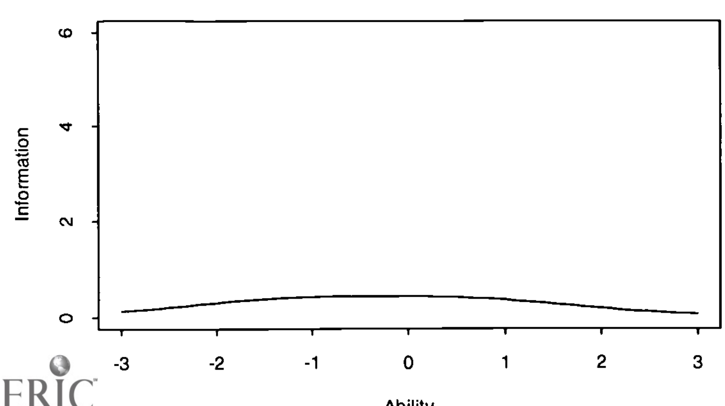

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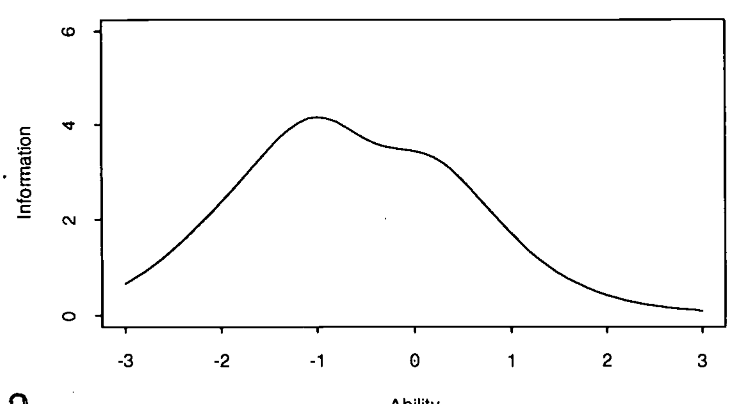

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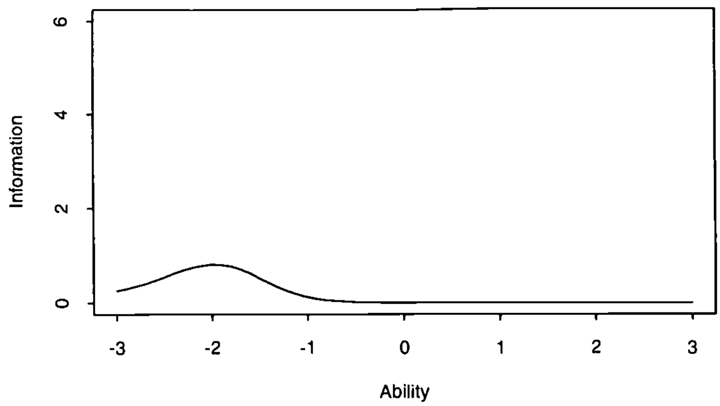

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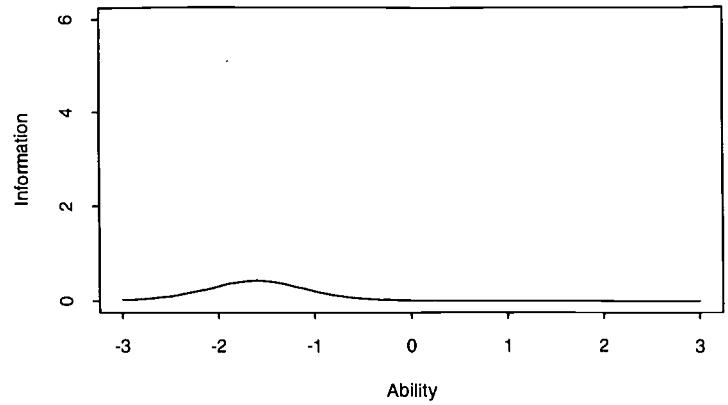

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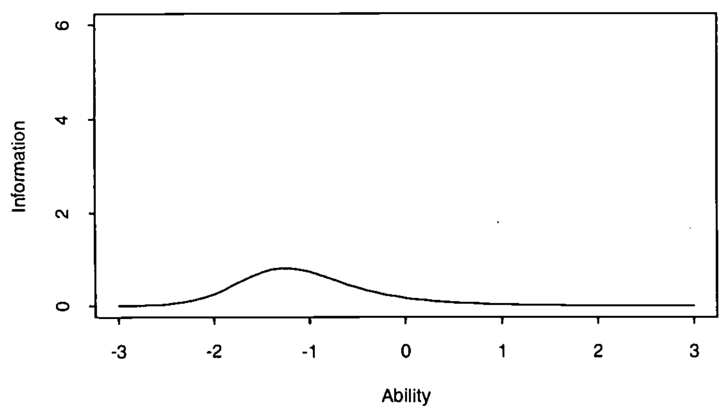

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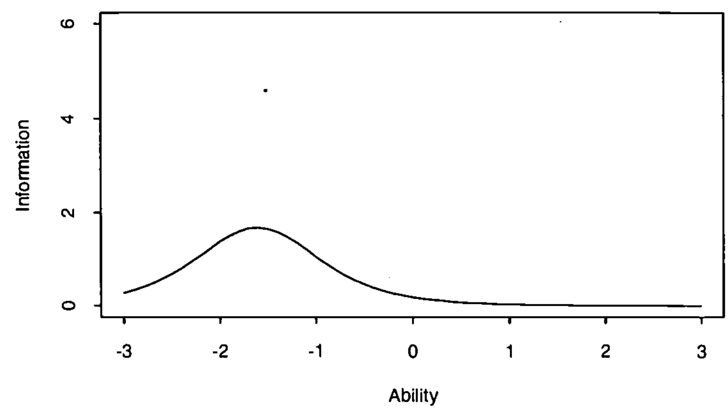

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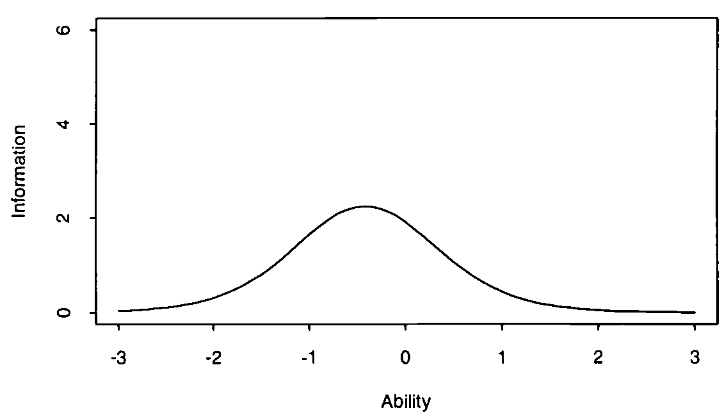

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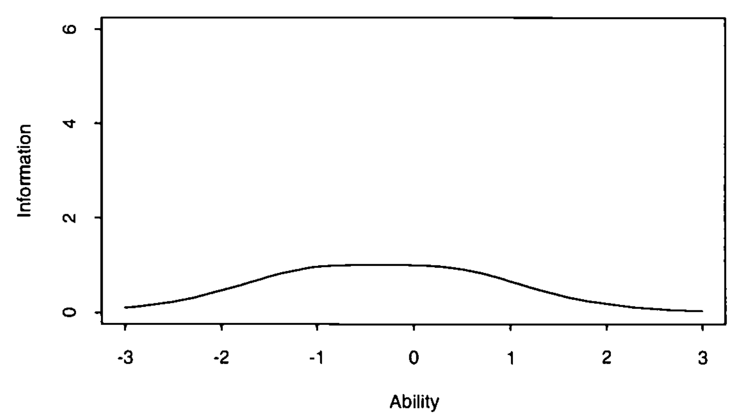

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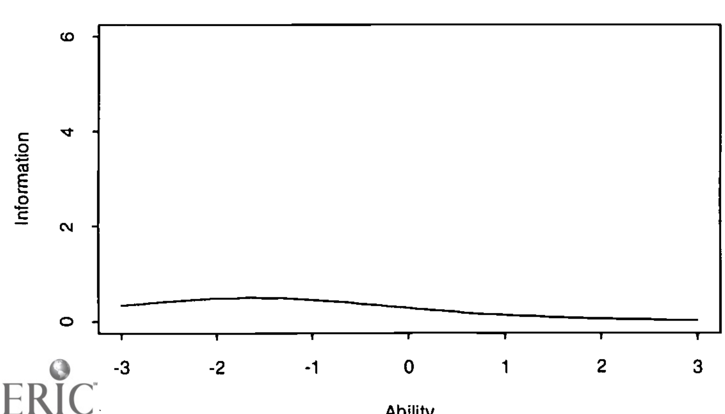

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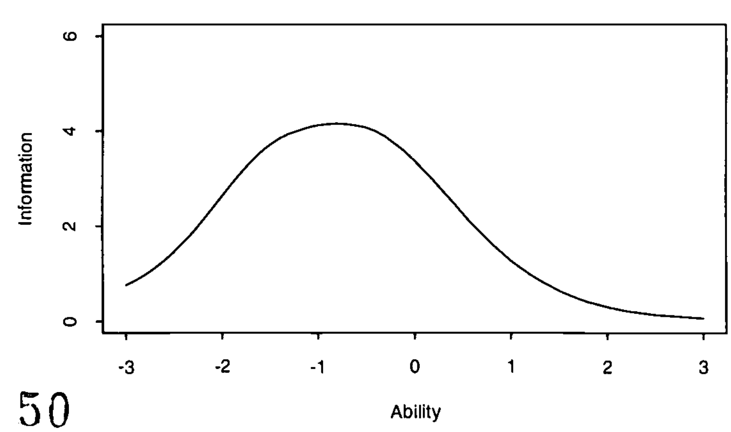

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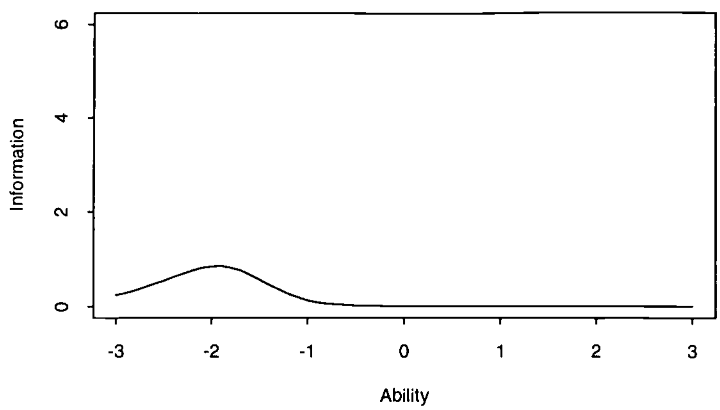

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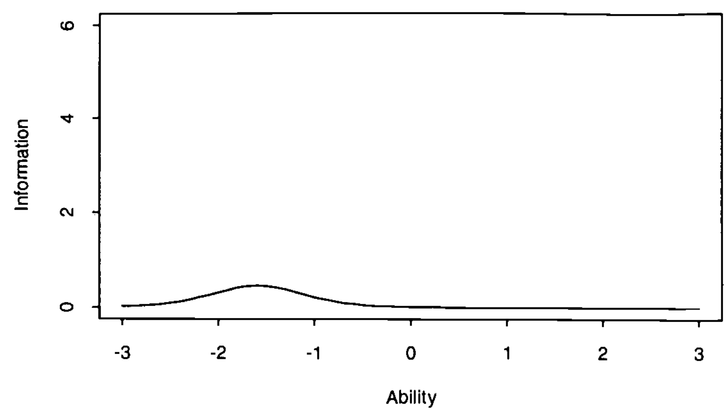

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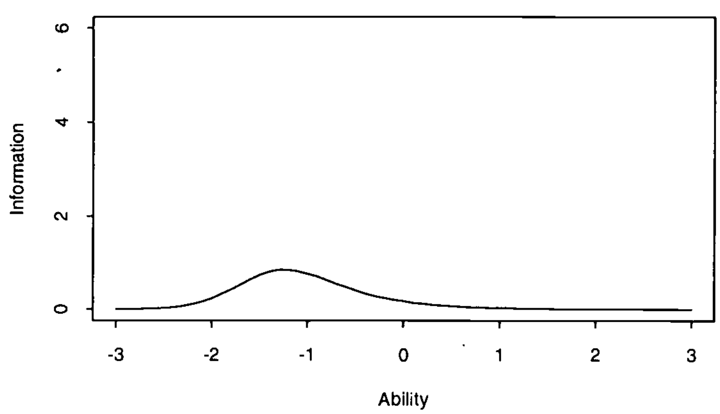

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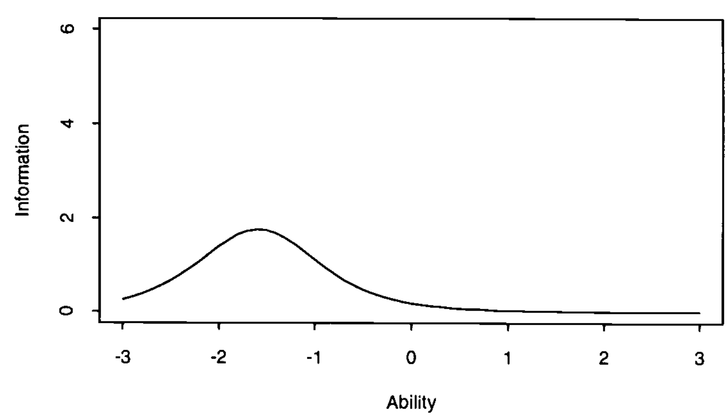

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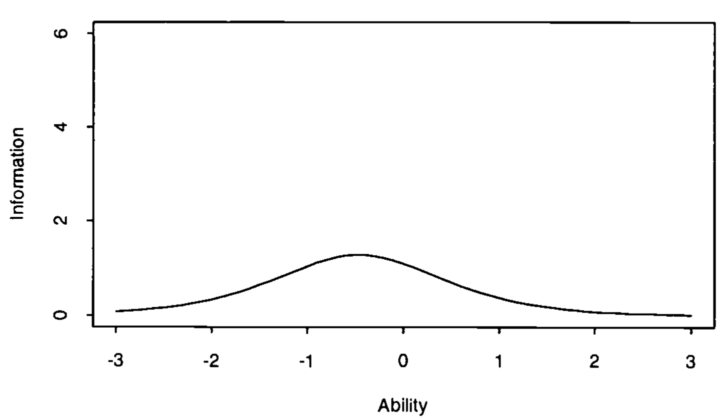

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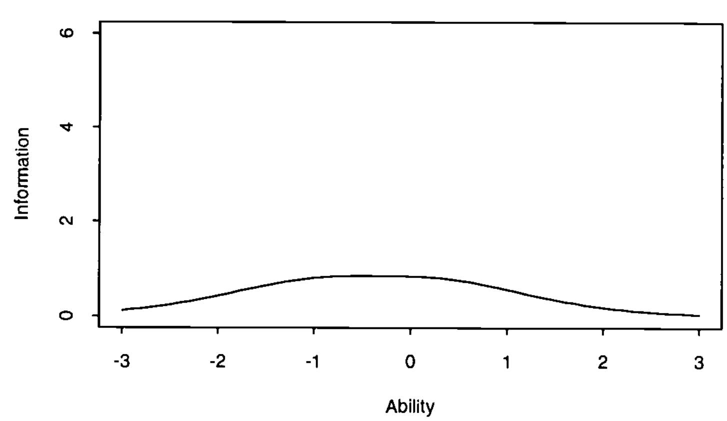

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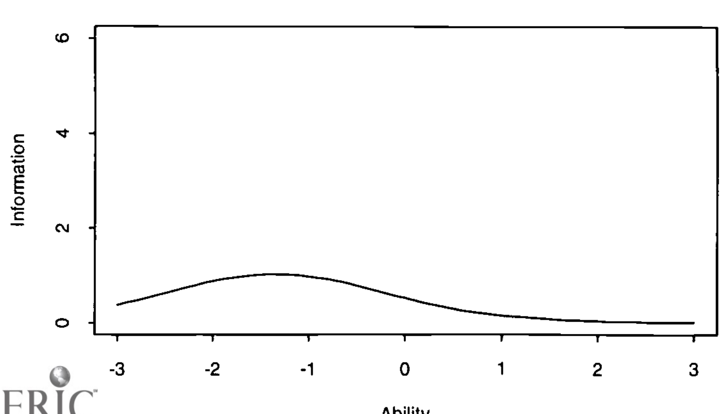

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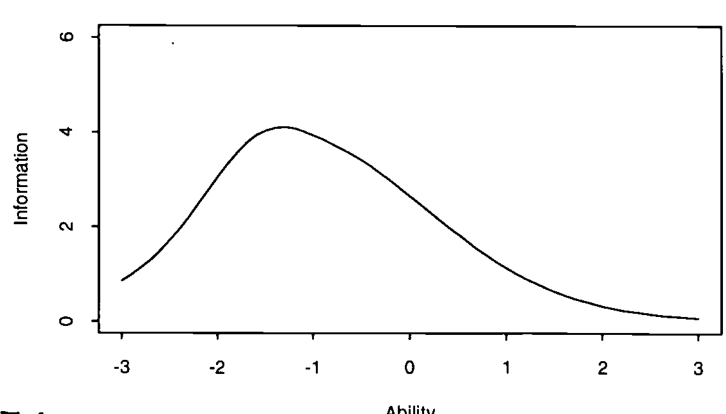

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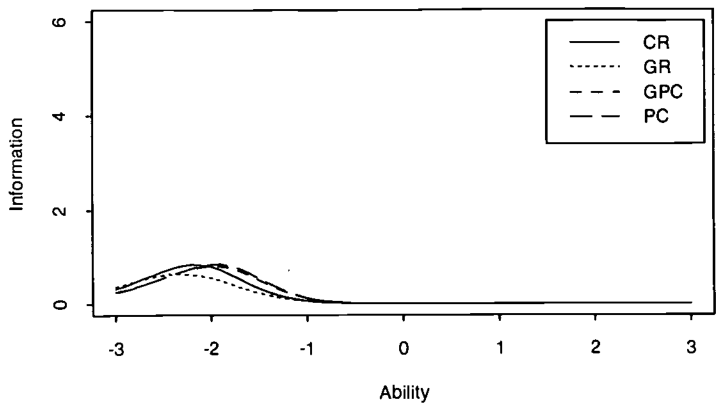

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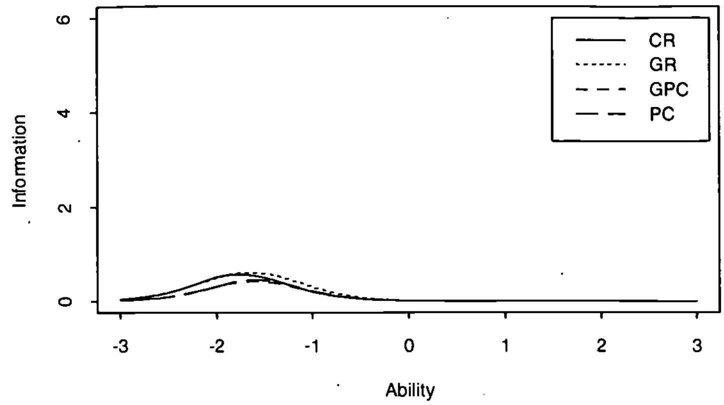

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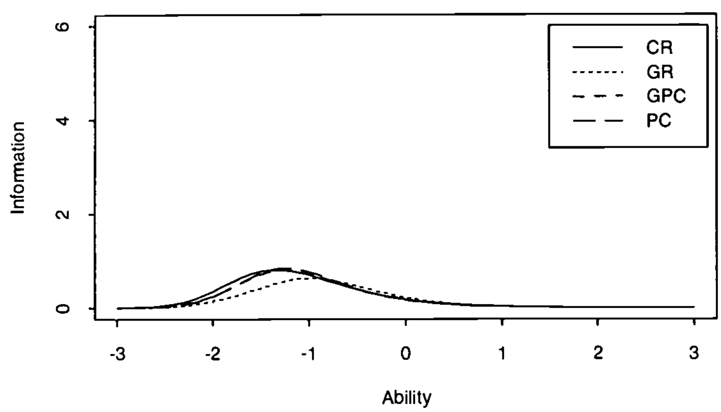

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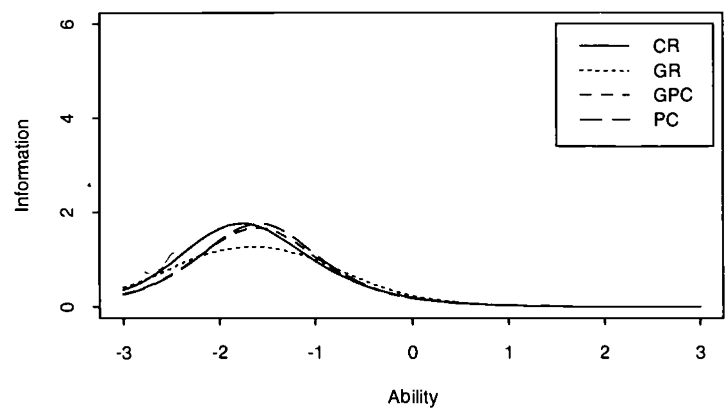

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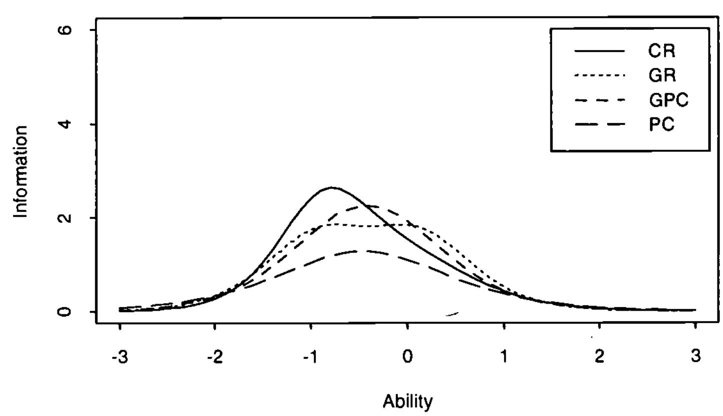

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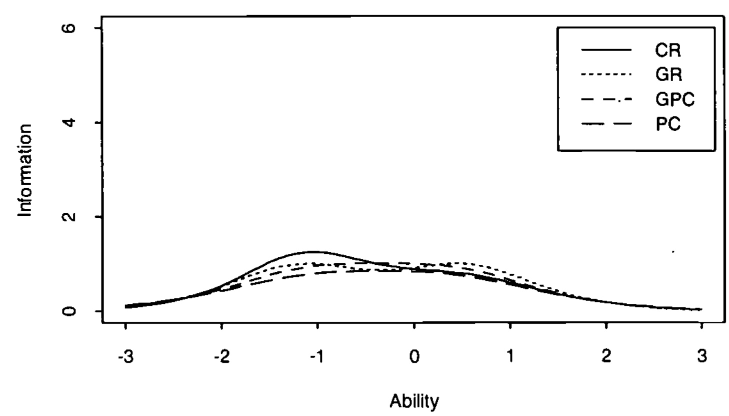

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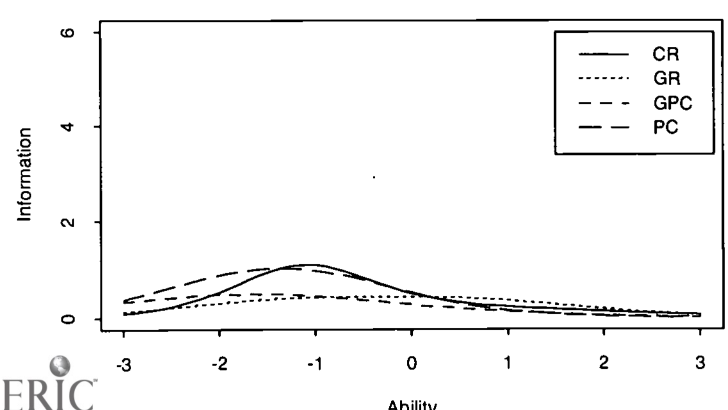

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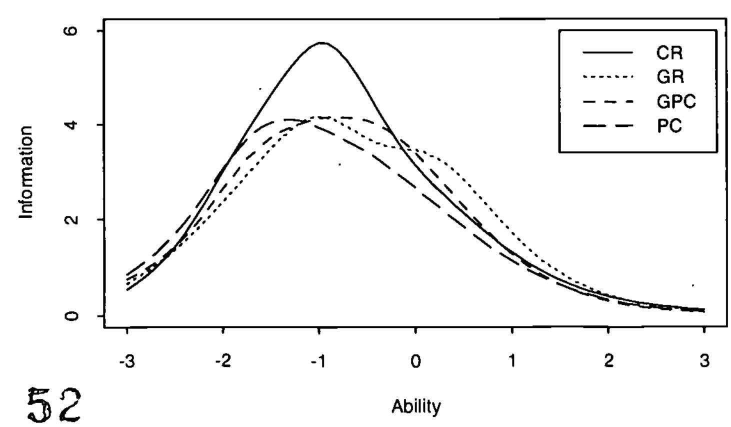

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